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"""
Registration: xxxx;
Description: Orthonormality and recursion relation for Legendre functions
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import numpy as np
from scipy.special import legendre
from scipy.misc import derivative
import scipy.integrate as sci
import warnings
warnings.filterwarnings("ignore")

# Feed the value of n, m, lower and upper limit, #of points (from keyboard)
n = 4; m = 5; start = -1; stop = 1; Np = 1000000;
x = np.linspace(start, stop, Np);

# Create Poly1D Legendre polynomial and derivatives
pn = legendre(n); pm = legendre(m); pnm1 = legendre(n-1);
pnm2 = legendre(n-2); pnp1 = legendre(n+1);
pnprime = derivative(pn, x, 1e-6) # spacing=10^-6
pnm1prime = derivative(pnm1, x, 1e-6)

# Logical case switch for different recursion relations to choice from
recl1=1; recl2=1; recl3=1; recl4=1; recl5=1; recl6=1;
print('Compare maximum of |lhs-rhs| (L1 norm) to zero for n = ', n)

if(recl1): # \int Pn(x)Pm(x) = 2/(2n+1)*delta(nm)
    I = sci.simps(pn(x)*pm(x),x)*(2.0*n+1)/2.0
    print('Orthonormality : \int P_2(x)P_2(x)dx = ', I)

if(recl2): #nPn(x) = (2n-1)xP(n-1)(x) - (n-1)P(n-2)(x)
    lhs = n*pn(x)
    rhs = (2*n-1)*x*pnm1(x)-(n-1)*pnm2(x)
    print('Maximum of nPn(x)-(2n-1)xPn(x)+(n-1)P(n-2)(x) = ', abs(max(lhs-rhs)))

if(recl3): #(n+1)P(n+1)(x) = (2n+1)xPn(x) - nP(n-1)(x)
    lhs = (n+1)*pnp1(x)
    rhs = (2*n+1)*x*pn(x)-n*pnm1(x)
    print('Maximum of (n+1)P(n+1)(x)-(2n+1)xPn(x)+nP(n-1)(x) = ', abs(max(lhs-rhs)))

if(recl4): #(1-x^2)Pn'(x) = n(P(n-1)(x) - xPn(x))
    lhs = (1-x**2)*pnprime
    rhs = n*(pnm1(x)-x*pn(x))
    print('Maximum of (1-x^2)dPn(x)/dx-n[P(n-1)(x)-xPn(x)] = ', abs(max(lhs-rhs)))

if(recl5): #nPn(x) = xPn'(x) - P(n-1)'(x)
    lhs = n*pn(x)
    rhs = x*pnprime-pnm1prime
    print('Maximum of nPn(x)-xdPn(x)/dx+dP(n-1)(x)/dx = ', abs(max(lhs-rhs)))

if(recl6): #Pn'(x) = xP(n-1)'(x) + nP(n-1)(x)
    lhs = pnprime
    rhs = x*pnm1prime + n*pnm1(x)
    print('Maximum of dPn(x)/dx-xdP(n-1)(x)/dx-nP(n-1)(x) = ', abs(max(lhs-rhs)))

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Results (2 Sets) :
Compare maximum of |lhs-rhs| (L1 norm) to zero for n = 2
Orthonormality : \int P_2(x)P_2(x)dx = 1.0
Maximum of nPn(x)-(2n-1)xPn(x)+(n-1)P(n-2)(x) = 1.11022302463e-15
Maximum of (n+1)P(n+1)(x)-(2n+1)xPn(x)+nP(n-1)(x) = 1.7763568394e-15
Maximum of (1-x^2)dPn(x)/dx-n[P(n-1)(x)-xPn(x)] = 1.6266898939e-10
Maximum of nPn(x)-xdPn(x)/dx+dP(n-1)(x)/dx = 3.68844066401e-10
Maximum of dPn(x)/dx-xdP(n-1)(x)/dx-nP(n-1)(x) = 3.71586317272e-10

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Compare maximum of |lhs-rhs| (L1 norm) to zero for n = 4
Orthonormality : \int P_4(x)P_5(x)dx = -1.45422004516e-16
Maximum of nPn(x)-(2n-1)xPn(x)+(n-1)P(n-2)(x) = 3.10862446895e-15
Maximum of (n+1)P(n+1)(x)-(2n+1)xPn(x)+nP(n-1)(x) = 4.4408920985e-15
Maximum of (1-x^2)dPn(x)/dx-n[P(n-1)(x)-xPn(x)] = 2.81584533468e-10
Maximum of nPn(x)-xdPn(x)/dx+dP(n-1)(x)/dx = 7.97428789667e-10
Maximum of dPn(x)/dx-xdP(n-1)(x)/dx-nP(n-1)(x) = 8.11806621925e-10
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