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"""
Registration : xxxx
Description  : BVP  $y'' = 12x^2$ ;  $y(0)=0$ ;  $y(1)=0$ ; Exactsol  $y = x^4-x$ 
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"""

import numpy as np
from scipy import sparse
import matplotlib.pyplot as plt

# Feed system size and fixed step-size
L = 50; dx = 1.0/L

# Construct finite (central) difference A matrix and b
x = np.linspace(0, 1, L+1).transpose()
b = np.zeros((L+1, 1)).ravel()
b[1:L] = 12 * pow(dx,2) * (x[1:L]**2)
main_diag = -2*np.ones((L+1,1)).ravel()
off_diag = 1*np.ones((L, 1)).ravel()
a = main_diag.shape[0]
diagonals = [main_diag, off_diag, off_diag]
A = sparse.diags(diagonals, [0,-1,1], shape=(a,a)).toarray()

# Enforce  $y(1) = y(L+1) = 0$ 
A[0, 0] = 1
A[0, 1] = 0
A[L, L] = 1
A[L, L-1] = 0

# Solve  $y = A^{-1} * b$ 
y = np.linalg.solve(A,b)

# To match with exact solution
xf = np.linspace(0, 1, 10*L)
yexact = pow(xf,4) - xf

# Plot
plt.figure(1);
plt.plot(x, y, 'ro', label='FD approximation', lw=3, ms=8)
plt.plot(xf, yexact, 'k-', label=r'Exact Solution  $y=x^4-x$ ', lw=3, ms=8)
plt.legend(loc='best', prop={'size':18})
plt.title(r'BVP using FD Method :  $\frac{d^2y}{dx^2} = 12x^2$ ',size=18)
plt.xlabel('x', size = 16); plt.xticks(size = 14);
plt.ylabel('y', size = 16); plt.yticks(size = 14)
plt.grid();
plt.savefig('plot/01_bvp.pdf')
plt.tight_layout()
plt.show()

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