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"""
Registration : xxxx
Description  : Raw-moments and cumulants of different distribution
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"""

import numpy as np
from scipy.special import comb
from scipy.stats import moment, kstat, describe

def rawmoment(N,n=1):
    #Calculate raw moments by using recursion from central moments
    mu = np.mean(N)
    mun = 0
    for i in range(0,n+1):
        mun += comb(n,i)*moment(N,i)*np.power(mu,n-i)
    return mun

def cumulant(N,n=1):
    #Calculate the cumulant using the cumulant matrix, given in Eqn2.28 in Risken, Chap.2
    K = np.zeros((n,n))
    for i in range(n):
        K[i,0] = rawmoment(N,i+1)
        for j in range(1,i+1,1):
            K[i,j] = comb(i,j-1)*rawmoment(N,i-j+1)
        if (i != n-1): K[i,i+1] = 1.0
    Kn = np.power(-1,n-1)*np.linalg.det(K)
    return Kn

#Binomial Distribution
n,p = 20,0.5
N = np.random.binomial(20,0.5,100000)
print ('Binomial Distribution')
print ('Mean, Variance, Skewness and Kurtosis are: ', describe(N).mean, ',',
describe(N).variance,',', describe(N).skewness, 'and ', describe(N).kurtosis)
print ('The first four raw moments are: ',rawmoment(N,1),',',rawmoment(N,
2),',',rawmoment(N,3),'and ',rawmoment(N,4))
print ('The theoretical value of the first four raw moments are: ',n*p,',',n*p*(1-
p+n*p),',',n*p*(1-3*p + 3*n*p + 2*p**2 - 3*n*p**2 + n**2*p**2),'and ',n*p*(1-7*p + 7*n*p +
12*p**2 - 18*n*p**2 + 6*n**2*p**2-6*p**3+11*n*p**3-6*n**2*p**3+n**3*p**3))
print ('The first four central moments are: ',moment(N,1),',',moment(N,2),',',moment(N,
3),'and ',moment(N,4))
print ('The theoretical value of the first four central moments are: ',0.0,',',n*p*(1-
p),',',n*p*(1-p)*(1-2*p),'and ',n*p*(1-p)*(1+(3*n-6)*p*(1-p)))
print ('The first four cumulants are: ',kstat(N,1),',',kstat(N,2),',',kstat(N,3),'and
',kstat(N,4))
print ('The theoretical value of the first four cumulants are: ',n*p,',',n*p*(1-
p),',',n*p*(1-p)*(1-2*p),'and ', n*p*(1-p)*(1-6*p + 6*p**2))


#Poisson Distribution
lam = 4.0
N = np.random.poisson(lam,100000)
print ('\n\nPoisson Distribution')
print ('Mean, Variance, Skewness and Kurtosis are: ', describe(N).mean, ',',
describe(N).variance,',', describe(N).skewness, 'and ', describe(N).kurtosis)
print ('The first four moments are: ',rawmoment(N,1),',',rawmoment(N,2),',',rawmoment(N,
3),'and ',rawmoment(N,4))
print ('The theoretical value of the first four moments are:
',lam,',',lam*(1+lam),',',lam*(1+3*lam+lam**2),',',lam*(1+7*lam+6*lam**2+lam**3))
print ('The first four central moments are: ',moment(N,1),',',moment(N,2),',',moment(N,
3),'and ',moment(N,4))
print ('The theoretical value of the first four central moments are: ',
0,',',lam,',',lam,',',lam*(1+3*lam))
print ('The first four cumulants are: ',kstat(N,1),',',kstat(N,2),',',kstat(N,3),'and
',kstat(N,4))
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print ('The theoretical value of the first four cumulants is: ',lam)

#Normal Distribution
mu,sig = 2.0,0.1
N = np.random.normal(mu,sig,100000)
print ('\n\nNormal Distribution')
print ('Mean, Variance, Skewness and Kurtosis are: ', describe(N).mean, ',',
describe(N).variance,',', describe(N).skewness, 'and ', describe(N).kurtosis)
print ('The first four moments are: ',rawmoment(N,1),',',rawmoment(N,2),',',rawmoment(N,
3),'and ',rawmoment(N,4))
print ('The theoretical value of the first four moments are: ',mu,',',mu**2 +
sig**2,',',mu**3 + 3*mu*sig**2,'and ',mu**4 + 6*mu**2*sig**2 + 3*sig**4)
print ('The first four central moments are: ',moment(N,1),',',moment(N,2),',',moment(N,
3),'and ',moment(N,4))
print ('The theoretical value of the first four central moments are: ',0.0,',',sig**2,',',
0.0,'and ',3*sig**4)
print ('The first four cumulants are: ',kstat(N,1),',',kstat(N,2),',',kstat(N,3),'and
',kstat(N,4))
print ('The theoretical value of the first four cumulants are: ',mu,',',sig**2,',',
0.0,'and ',0.0)

#Uniform Distribution
a,b = -1,0
N = np.random.uniform(a,b,100000)
print ('\n\nUniform Distribution')
print ('Mean, Variance, Skewness and Kurtosis are: ', describe(N).mean, ',',
describe(N).variance,',', describe(N).skewness, 'and ', describe(N).kurtosis)
print ('The first four moments are: ',rawmoment(N,1),',',rawmoment(N,2),',',rawmoment(N,
3),'and ',rawmoment(N,4))
print ('The theoretical value of the first four moments are: ',0.5*(a+b),',',
(1/3.0)*(a**2+b**2+a*b),',',(1/4.0)*(a+b)*(a**2+b**2),'and ',(1/5.0)*(a**4 + b**4 + a**3*b
+ a*b**3 + a**2*b**2))
print ('The first four central moments are: ',moment(N,1),',',moment(N,2),',',moment(N,
3),'and ',moment(N,4))
print ('The theoretical value of the first four central moments are: ',0.0,',',
(1/12.0)*(b-a)**2,',',0.0,'and ',(1/80.0)*(b-a)**4)
print ('The first four cumulants are: ',kstat(N,1),',',kstat(N,2),',',kstat(N,3),'and
',kstat(N,4))
print ('The theoretical value of the first four cumulants are: ',-0.5,',',1/12.0,',',
0.0,'and ',-1/120.0)

#Exponential Distribution
lam = 1.5
N = np.random.exponential((1.0/lam),100000)
print ('\n\nExponential Distribution')
print ('Mean, Variance, Skewness and Kurtosis are: ', describe(N).mean, ',',
describe(N).variance,',', describe(N).skewness, 'and ', describe(N).kurtosis)
print ('The first four moments are: ',rawmoment(N,1),',',rawmoment(N,2),',',rawmoment(N,
3),'and ',rawmoment(N,4))
print ('The theoretical value of the first four moments are: ',(1.0/lam),',',(2.0/
(lam**2)),',',(6.0/(lam**3)),'and ',(24.0/(lam**4)))
print ('The first four central moments are: ',moment(N,1),',',moment(N,2),',',moment(N,
3),'and ',moment(N,4))
print ('The theoretical value of the first four central moments are: ',0,',',(1.0/
(lam**2)),',',(2.0/(lam**3)),'and ',(9.0/(lam**4)))
print ('The first four cumulants are: ',kstat(N,1),',',kstat(N,2),',',kstat(N,3),'and
',kstat(N,4))
print ('The theoretical value of the first four cumulants are: ',(1.0/lam),',',(1.0/
(lam**2)),',',(2.0/(lam**3)),'and ',(6.0/(lam**4)))


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Results:

Binomial Distribution
Mean, Variance, Skewness and Kurtosis are:  10.00551 , 5.005169691596915 ,
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-0.0003058530935106756 and  -0.1119942977189643
The first four raw moments are:  10.00551 , 105.11534999999999 , 1151.8868099999997 and
13100.651309999996
The theoretical value of the first four raw moments are:  10.0 , 105.0 , 1150.0 and
13072.5
The first four central moments are:  0.0 , 5.005119639899999 , -0.003424794931686365 and
72.34807374596188
The theoretical value of the first four central moments are:  0.0 , 5.0 , 0.0 and  72.5
The first four cumulants are:  10.00551 , 5.005169691596916 , -0.0034248976777013513 and
-2.804287319888237
The theoretical value of the first four cumulants are:  10.0 , 5.0 , 0.0 and  -2.5


Poisson Distribution
Mean, Variance, Skewness and Kurtosis are:  3.99589 , 3.9890929988299884 ,
0.5033065886331817 and  0.2767901063285927
The first four moments are:  3.99589 , 19.95619 , 115.63231000000002 and
753.3471099999999
The theoretical value of the first four moments are:  4.0 , 20.0 , 116.0 , 756.0
The first four central moments are:  0.0 , 3.9890531079000002 , 4.009935094246936 and
52.14206903176005
The theoretical value of the first four central moments are:  0 , 4.0 , 4.0 , 52.0
The first four cumulants are:  3.99589 , 3.9890929988299884 , 4.0100055395106786 and
4.40569807288962
The theoretical value of the first four cumulants is:  4.0


Normal Distribution
Mean, Variance, Skewness and Kurtosis are:  2.0000938280810594 , 0.01002826859296044 ,
0.001670377516946261 and  0.0032740881244985864
The first four moments are:  2.0000938280810594 , 4.010403489438221 , 8.061299499869047
and  16.244016773923153
The theoretical value of the first four moments are:  2.0 , 4.01 , 8.06 and
16.240299999999998
The first four central moments are:  0.0 , 0.01002816831027451 , 1.6774402415684264e-06
and  0.0003020217348984721
The theoretical value of the first four central moments are:  0.0 , 0.010000000000000002 ,
0.0 and  0.0003000000000000003
The first four cumulants are:  2.0000938280810594 , 0.010028268592961197 ,
1.6774905653814634e-06 and  3.353131865055478e-07
The theoretical value of the first four cumulants are:  2.0 , 0.010000000000000002 , 0.0
and  0.0


Uniform Distribution
Mean, Variance, Skewness and Kurtosis are:  -0.5008569113617904 , 0.08289642029689884 ,
0.004051767664968396 and  -1.1960193354783624
The first four moments are:  -0.5008569113617904 , 0.3337532369915683 ,
-0.2501035717304549 and  0.19990215357137567
The theoretical value of the first four moments are:  -0.5 , 0.3333333333333333 , -0.25
and  0.2
The first four central moments are:  0.0 , 0.08289559133269588 , 9.670338694818176e-05
and  0.012396376161362948
The theoretical value of the first four central moments are:  0.0 , 0.08333333333333333 ,
0.0 and  0.0125
The first four cumulants are:  -0.5008569113617904 , 0.08289642029689884 ,
9.6706288117745851e-05 and  -0.008218824032096771
The theoretical value of the first four cumulants are:  -0.5 , 0.08333333333333333 , 0.0
and  -0.008333333333333333


Exponential Distribution
Mean, Variance, Skewness and Kurtosis are:  0.6675561205470499 , 0.4447753996514606 ,
1.9921440915425497 and  5.908627430005433
The first four moments are:  0.6675561205470499 , 0.8904021259772916 , 1.7791280166589032
and  4.728002313604621
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The theoretical value of the first four moments are:  0.6666666666666666 ,
0.8888888888888888 , 1.7777777777777777 and  4.7407407407407405
The first four central moments are:  0.0 , 0.44477095189746413 , 0.5909154853532773 and
1.7623153654543962
The theoretical value of the first four central moments are:  0 , 0.4444444444444444 ,
0.5925925925925926 and  1.7777777777777777
The first four cumulants are:  0.6675561205470499 , 0.4447753996514607 ,
0.5909332132314875 and  1.1689454597304634
The theoretical value of the first four cumulants are:  0.6666666666666666 ,
0.4444444444444444 , 0.5925925925925926 and  1.1851851851851851
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