# Evolution of the magnetic field in a Quark Star

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## 1 Magnetic fields of Neutron Stars

A white dwarf or a neutron star or a black hole is left behind when a star completely exhausts its nuclear fuel. These objects do not have an active source of energy and hence can not support themselves against gravitational collapse by thermal pressure. Instead, the outward pressure is due to degenerate electrons in white dwarfs and mostly due to degenerate neutrons in neutron stars. The neutron stars manifest themselves as radio pulsars and also as the compact object in many X - ray binaries. Recently, some of these objects are being suspected to be Strange Quark stars rather than neutron stars. In this work we investigate the evolution of the magnetic field in Strange stars, in order to understand the possible differences between a neutron star and a strange star in this context.

Neutron stars are born in the supernova explosion in the last phases of evolution of a massive star. Typically a neutron star has a mass $\sim 1 - 2M_{\odot}$, radius of $\sim$ 10 Km and consequently an average density of $10^{15} gmcm^{-3}$. Observation of radio pulsars have also revealed that they have magnetic fields ranging from $10^8$ G to $10^{15}$ G and spin periods in the interval $\sim$ 1.5 ms to $10^2$ s.

It has been observed that the magnetic field of a neutron star is dipolar in nature. Though there are controversies, there are two main theories for the generation of this huge magnetic field. The field can either be a fossil remnant from a progenitor star or can be generated after the formation of the neutron star due to thermo - magnetic instabilities. The main dissipation mechanism of the magnetic field is ohmic decay because of the very large electrical conductivity. Observationally, it has been found that -

- Isolated pulsars with high magnetic fields ($\sim 10^{11} - 10^{13.5}$ G) do not undergo any significant field decay during their lifetimes.
- The fact that binary pulsars, as well as millisecond and globular cluster pulsars which almost always have a binary history, possess much lower field strengths suggests that significant field decay occurs only as a result of the interaction of a neutron star with its binary companion.

(See Shapiro & Teukolsky (1983) for the details of neutron star physics and Konar (1997) for a review of the pulsar magnetic field phenomenology.)

## 2 Physics of Strange Stars

*Strange Quark Matter* (SQM), composed of u, d and s quarks, may probably be the ultimate ground state of matter Witten (1984). If meta-stable at zero pressure this phase might exist in the central region of a compact object (white dwarfs, neutron stars) stabilized by the high pressure. If however, SQM is absolutely stable at zero pressure the existence of *Strange Stars* (with or without a thin hadronic crust) is a possibility. It has been found that the stable range of mass ($1\mathrm{M}_{\odot} - 2\mathrm{M}_{\odot}$) for strange stars is quite similar to that for neutron stars. Furthermore, in this range the radii of strange stars are not very different from those of the standard neutron stars. Since, the range of stable rotation periods sustainable by these two types of stars are obviously similar, there has been speculations that perhaps some or all of the pulsars are strange stars instead of neutron stars. (See Glendenning (1997) for details of Strange Star physics & phenomenology.)

Considerable effort has been spent to distinguish strange stars from neutron stars observationally. However, as yet there is no definite observational characteristic to distinguish a strange star from a neutron star (see Konar (2000) for a brief discussion ). Surprisingly, even though one of the most important feature of a neutron star is its magnetic field, there has been little work done to compare the nature of the magnetic field in a neutron star with that in a strange star. In the present work, we investigate the nature of the evolution of the magnetic field in a strange star. This is done with an aim to compare this with the generic nature of the evolution of the magnetic field in the neutron stars and see if there are any telltale differences.

# 3 Physics of Magnetic Field Evolution

## 3.1 One-Component Plasma

In a plasma with only one kind of charged particles, the Maxwell's equations are (Jackson, 1975)

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}, \tag{1}$$

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0; \tag{2}$$

where **E**, **B**, **J** are the electric field, the magnetic field and the current density respectively. In a homogeneous and isotropic system, the generalized *Ohm's law* is given by

$$\mathbf{J} = \sigma(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}). \tag{3}$$

Combining the Maxwell's equations with generalized Ohm's law we obtain the following *induction equation* giving the time evolution of the magnetic field in a one-component plasma -

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \frac{c^2}{4\pi}\nabla \times \left(\frac{1}{\sigma}\nabla \times \mathbf{B}\right) \tag{4}$$

where $\mathbf{v}$ is the drift velocity of the charged species and $\sigma$ is the electrical conductivity of the plasma.

## 3.2 Two-Component Plasma

The evolution of the magnetic field in an $n - p - e$ plasma, in the core of a neutron star, has been studied extensively by Goldreich & Reisenegger (1992). In this section we review that work. The interior of a neutron star is made up of an $n - p - e$ plasma with all the components being Fermi-degenerate. In the present work we assume the protons and the neutrons to be in a degenerate but non-superfluid state. There are two types of charge carriers - the electrons and the protons. The neutrons, being neutral, are assumed to provide the stationary background. Furthermore, since they are heavy, it's assumed that the relaxation time of neutron-proton, neutron-electron scattering are very large so that the terms containing $1/\tau_{pn}$ can be safely neglected without losing any generality.

The local state of each particle species is described by its chemical potential $\mu_i$. It is assumed that weak interactions tend to erase the perturbations away from chemical equilibrium (through inverse $\beta$-capture and $\beta$-decay) and the drag forces due to elastic binary collisions impede the relative motions of the different particle species. In presence of a magnetic field, the charged particles cannot be in simultaneous magneto-static and chemical equilibrium with the neutrons. Therefore, the differential equations of motion for each species can be set up which when solved give the equation for the evolution of the magnetic field.

The neutrons are assumed to provide a fixed background in diffusive equilibrium. The magnetic pressure is small compared to the particle pressure implying that the magnetic field cannot induce significant changes in the charge configuration of the fluid. The density profile of neutrons is given by:

$$\mu_n + m_n\psi = constant, \tag{5}$$

where, $\psi$ is the Newtonian gravitational potential at a particular point inside the star. Here, $\mu_n$ and $m_n$ denote the chemical potential and the mass of the neutron, respectively. Contributions to $\psi$ due to the protons and the electrons are neglected because of the relative smallness of their number density ($1 : 10$ compared to the neutrons). The equations of motion for the charged particles are:

$$m_p\frac{\partial \mathbf{v_p}}{\partial t} + m_p(\mathbf{v_p} \cdot \nabla)\mathbf{v_p} = -\nabla\mu_p - m_p\nabla\psi + e(\mathbf{E} + \frac{\mathbf{v_p}}{c} \times \mathbf{B})$$

$$-\frac{m_p \mathbf{v_p}}{\tau_{pn}} - \frac{m_p(\mathbf{v_p} - \mathbf{v_e})}{\tau_{pe}} \tag{6}$$

$$m_e^* \frac{\partial \mathbf{v_e}}{\partial t} + m_e^*(\mathbf{v_e} \cdot \nabla)\mathbf{v_e} = -\nabla \mu_e - e(\mathbf{E} + \frac{\mathbf{v_e}}{c} \times \mathbf{B}) - \frac{m_e^* \mathbf{v_e}}{\tau_{en}} - \frac{m_e^*(\mathbf{v_e} - \mathbf{v_p})}{\tau_{ep}} \tag{7}$$

Here, $m_e^* = \mu_e/c^2$ is defined as the effective inertia of the electrons, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields, $\mathbf{v_i}$ is the mean velocity of the species i and $\tau_{ij}$ are the relaxations time of species i with respect to species j. The average velocity of neutrons are assumed to be zero. We ignore all the relativistic corrections to the inertia of the neutrons and protons. Conservation of momentum yields that $m_p/\tau_{pe} = m_e^*/\tau_{ep}$ The process involves small velocities that change over time scales much longer than the relaxation times. So neglecting the acceleration terms from the left sides of the above equations, we arrive at

$$\frac{\mathbf{f_B}}{n_c} - \nabla(\Delta\mu) = \frac{m_p \mathbf{v_p}}{\tau_{pn}} + \frac{m_e^* \mathbf{v_e}}{\tau_{en}} \equiv (\frac{m_p}{\tau_{pn}} + \frac{m_e^*}{\tau_{en}})\mathbf{v} \tag{8}$$

where $\Delta\mu \equiv \mu_p + \mu_e - \mu_n$ is the deviation from chemical equilibrium, $n_c \approx n_p \approx n_e$ is the number density of charged species and $\mathbf{f_B}$ is the magnetic force density given as

$$\mathbf{f_B} = \frac{\mathbf{J} \times \mathbf{B}}{c} \tag{9}$$

where the current density $\mathbf{J}$ is defined as

$$\mathbf{J} \equiv e n_c(\mathbf{v_p} - \mathbf{v_e}) \tag{10}$$

Equation (8) suggests that magneto-static equilibrium requires $\mathbf{f_B}/n_c$ to be the gradient of a potential. Only then the gradient of the perturbed chemical potential balance the magnetic force density. If this does not apply, then forces drive the charged particles through the fixed background of neutrons at the ambipolar diffusion velocity $\mathbf{v}$. From equations (6) & (7) it is found that

$$\mathbf{E} = \frac{\mathbf{J}}{\sigma_0} - \frac{\mathbf{v}}{c} \times \mathbf{B} + \left(\frac{m_p/\tau_{pn} - m_e^*/\tau_{en}}{m_p/\tau_{pn} + m_e^*/\tau_{en}}\right) \frac{\mathbf{J} \times \mathbf{B}}{n_c e c} + \frac{(\tau_{pn}/m_p)\nabla\mu_p - (\tau_{en}/m_e^*)\nabla\mu_e}{e(\tau_{pn}/m_p + \tau_{en}/m_e^*)}, \tag{11}$$

where

$$\sigma_0 = n_c e^2 \left(\frac{1}{\tau_{ep}/m_e^*} + \frac{1}{\tau_{pn}/m_p + \tau_{en}/m_e^*}\right)^{-1}. \tag{12}$$

Here $m_p, m_e$ denote the proton and the electron mass. $\tau_{ij}$ is the relaxation time for the $ij$ collision process and $n_c$ is the number density of a particular species of charged particle (same for protons and electrons to ensure charge neutrality).

Therefore, the evolution of magnetic field in this $n - p - e$ plasma is obtained to be

$$\begin{aligned} \frac{\partial B}{\partial t} &= -c\nabla \times \frac{\mathbf{J}}{\sigma_0} + \nabla \times (\mathbf{v} \times B) \\ &\quad - \frac{m_p/\tau_{pn} - m_e^*/\tau_{en}}{m_p/\tau_{pn} + m_e^*/\tau_{en}} \nabla \times (\frac{\mathbf{J} \times \mathbf{B}}{n_c e}), \end{aligned} \tag{13}$$

The induction equation shows that there are three mechanisms that corresponds to the decay of magnetic field, from an isolated namely the *Ohmic decay, Ambipolar diffusion* and *Hall drift*. Ohmic decay produces a diffusion of magnetic flux with respect to the charged particles. In general it is independent of the magnetic field and is inversely proportional to the electrical conductivity. Ambipolar diffusion generates a drift of the magnetic field and charged particles relative to a fixed background. The ambipolar drift velocity is decomposed into a solenoidal and an irrotational component due to the charged particle flux associated with the diffusion. This drift velocities are directly proportional to the square of the magnetic field strength if the charged components form a normal fluid model. This type of diffusion comes into picture only where the charge particle composition is inhomogeneous. The solenoidal component is responsible for transporting magnetic flux on a short time scale and the irrotational component perturbs the chemical equilibrium between the components. Hall drift relates to the Hall component of electric field. The drift velocity is directly proportional to the magnetic field strength. Hall drift is not itself responsible for magnetic field decay, in fact, Hall drift conserves magnetic energy as magnetic force is a no work one. However, it allows for mixing of various multipoles of the field.

We resolve $\mathbf{v}$ and $\mathbf{f_B}$ into solenoidal (divergence-free) and irrotational (curl-free) components, $\mathbf{v^s}$ and $\mathbf{f_B^s}$,and $\mathbf{v^{ir}}$ and $\mathbf{f_B^{ir}}$. Then

$$\mathbf{v^s} = \frac{\mathbf{f_B^s}}{n_c(m_p/\tau_{pn} + m_e^*/\tau_{en})} \tag{14}$$

$$\mathbf{v^{ir}} = \frac{\mathbf{f_B^{ir}} - n_c \nabla(\Delta\mu)}{n_c(m_p/\tau_{pn} + m_e^*/\tau_{en})} \tag{15}$$

The time scales for the ohmic decay is given from equation () as

$$t_{ohmic} \sim \frac{4\pi\sigma_0 L^2}{c^2} \tag{16}$$

Ohmic decay time scales is proportional to $L^2$ and independent of $\mathbf{B}$.
The other two time scales for the ambipolar diffusion is depicted below as the solenoidal and irrotational component

$$t^s_{ambip} \sim \frac{L}{\mathbf{v^s}} \sim \frac{4\pi n_c L^2}{B^2}(\frac{m_p}{\tau_{pn}} + \frac{m_e^*}{\tau_{en}}) \tag{17}$$

$$t^{ir}_{ambip} \sim \frac{L}{\mathbf{v^{ir}}} \sim \frac{4\pi n_c (L^2 + a^2)}{B^2}(\frac{m_p}{\tau_{pn}} + \frac{m_e^*}{\tau_{en}}) \tag{18}$$

# 4 The four component u - d - s - e plasma

Inside a quark star, because of quark - deconfinement we expect to get the four component $u-d-s-e$ plasma. Though quarks interact through the *strong interaction* they are asymptotically free in the density regime obtained in the interior of a neutron star or strange star. Therefore, we can study the electromagnetic interaction between these quarks in the limit of asymptotic freedom to study the behavior of the evolution of the magnetic field.

Each quark species is assumed to behave like an ideal, Fermi-degenerate gas. Like in the case of a neutron star interior, we specify the local state of each charged species by its chemical potential $\mu_i$. The strange quark is assumed to provide a stationary background in diffusive equilibrium because it is much heavier than the other two types of quarks. Since, $m_s >> m_u, m_d$ we can assume $\mathbf{v}_s = 0$. Therefore the equation of motions are

$$\begin{aligned} m_u \frac{d\mathbf{v_u}}{dt} + m_u(\mathbf{v_u}.\nabla).\mathbf{v_u} &= -\nabla\mu_u - m_u\nabla\psi + \frac{2}{3}e(\mathbf{E} + \frac{\mathbf{v_u}}{c} \times \mathbf{B}) \\ &\quad - \frac{m_u \mathbf{v_u}}{\tau_{us}} - \frac{m_u(\mathbf{v_u} - \mathbf{v_d})}{\tau_{ud}} \end{aligned} \tag{19}$$

$$\begin{aligned} m_d \frac{d\mathbf{v_d}}{dt} + m_d(\mathbf{v_d}.\nabla).\mathbf{v_d} &= -\nabla\mu_d - m_d\nabla\psi - \frac{e}{3}(\mathbf{E} + \frac{\mathbf{v_d}}{c} \times \mathbf{B}) \\ &\quad - \frac{m_d \mathbf{v_d}}{\tau_{ds}} - \frac{m_d(\mathbf{v_d} - \mathbf{v_u})}{\tau_{du}} \end{aligned} \tag{20}$$

$$0 = -\nabla\mu_s - m_s\nabla\psi - e\mathbf{E}/3 - \frac{m_s\mathbf{v_u}}{\tau_{su}} - \frac{m_s\mathbf{v_d}}{\tau_{sd}} \tag{21}$$

Charge neutrality demands that,

$$\frac{2}{3}n_u - \frac{1}{3}(n_d + n_s) - n_e = 0. \tag{22}$$

The generalized current density can be defined as

$$\mathbf{J} = \frac{e}{3}(2n_u\mathbf{v}_u - n_d\mathbf{v}_d - 3n_e\mathbf{v}_e). \tag{23}$$

Hence, neglecting the accelerations, we have,

$$-(1-x)\nabla\mu + \frac{2e}{3}(\mathbf{E} + \frac{\mathbf{v_u}}{c} \times \mathbf{B}) - m_u\Sigma_{i\neq u}\frac{\mathbf{v_u} - \mathbf{v_i}}{\tau_{ui}} = 0, \tag{24}$$

$$-\nabla\mu - \frac{e}{3}(\mathbf{E} + \frac{\mathbf{v_d}}{c} \times \mathbf{B}) - m_d\Sigma_{i\neq d}\frac{\mathbf{v_d} - \mathbf{v_i}}{\tau_{di}} = 0, \tag{25}$$

$$-x\nabla\mu - e(\mathbf{E} + \frac{\mathbf{v_e}}{c} \times \mathbf{B}) - m_e^*\Sigma_{i\neq e}\frac{\mathbf{v_e} - \mathbf{v_i}}{\tau_{ei}} = 0, \tag{26}$$

$$-\nabla\mu - m_s\nabla\psi - \frac{e}{3}\mathbf{E} = 0, \tag{27}$$

where, $m_e^* = \mu_e/c^2$, $\mathbf{v_i}$ is the mean velocity of the i-th species, $\tau_{ij}$ is the relaxation time of the $i$-th species with respect to $j$-th species. Conservation of momentum implies $m_i/\tau_{ij} = m_j/\tau_{ji}$. Also, $\mu_s = \mu_d = \mu$, $\mu_u = \mu(1-x)$, $\mu_e = \mu x$ (using the notation of Glendenning 1997). Therefore, an appropriate combination of the above equations gives,

$$-\nabla(\Delta\mu) - n_s m_s \nabla\psi + \frac{\mathbf{J}\times\mathbf{B}}{c} - \Sigma_{i\neq j,s}\frac{m_i}{\tau_{ij}}(n_i - n_j)\mathbf{v}_{\rm ij} - \left(\frac{n_u m_u}{\tau_{us}} + \frac{n_d m_d}{\tau_{ds}} + \frac{n_e m_e}{\tau_{es}}\right)\mathbf{v}_{\rm av} = 0, \tag{28}$$

where,

$$\Delta\mu = ((1-x)n_u + n_d + n_s + x n_e)\mu \tag{29}$$

$$\mathbf{v}_{\rm ij} = \mathbf{v}_i - \mathbf{v}_j \tag{30}$$

$$\mathbf{v}_{\rm av} = \left(\frac{n_u m_u}{\tau_{us}} + \frac{n_d m_d}{\tau_{ds}} + \frac{n_e m_e}{\tau_{es}}\right)^{-1} \times \left(\frac{n_u m_u}{\tau_{us}}\mathbf{v}_u + \frac{n_d m_d}{\tau_{ds}}\mathbf{v}_d + \frac{n_e m_e}{\tau_{es}}\mathbf{v}_e\right). \tag{31}$$

Adding equations (21), (22), (23) and (24) one can obtain

$$\mathbf{E} = \frac{2(\mathbf{v_u} - \mathbf{v_d} - \mathbf{3v_e})\times\mathbf{B}}{3c} - \frac{m_s\nabla\psi}{e} - \frac{3\nabla\mu}{e} - \frac{1}{e}\Sigma_{i\neq s}\frac{m_i\mathbf{v_i}}{\tau_{is}} \tag{32}$$

From equations (20) and (28) by simple algebra, one can find the velocities of the u and d quark in terms of $\mathbf{v_{av}}$, $\mathbf{J}$ and $\mathbf{v_e}$. Here it is assumed that $\mathbf{v_s} = 0$. We substitute $\nabla\mu$ from equation (25) & (26) in equation (29) and simplify to get the electric field vector in terms of $\mathbf{v_{av}}$, $\mathbf{J}$ and $\mathbf{v_e}$ :

$$\begin{aligned}\mathbf{E} = {} & \frac{2\Sigma_{i\neq j,s}\frac{m_i n_i}{\tau_{is}}(n_d - n_u)}{3cn_u n_d(m_u/\tau_{us} + 2m_d/\tau_{ds})}(\mathbf{v}_{\rm av}\times\mathbf{B}) \\ & + \left[\frac{3}{ec\Theta} + \frac{m_u n_u/\tau_{us} + 2m_d n_d/\tau_{ds}}{ecn_u n_d(m_u/\tau_{us} + 2m_d/\tau_{ds})}\right](\mathbf{J}\times\mathbf{B}) \\ & + \frac{n_e[3m_u n_u/\tau us + 6m_d n_d/\tau_{ds} + 2m_e(n_u - n_d)/\tau es] - 3n_u n_d}{3cn_u n_d(m_u/\tau_{us} + 2m_d/\tau_{ds})}(\mathbf{v_e}\times\mathbf{B}) \\ & + \frac{\Sigma_{i\neq s}m_i n_i/\tau_{is}[6X/\Theta - (m_u n_d/\tau_{us} + 2m_d n_u/\tau_{ds})]}{en_u n_d(m_u/\tau_{us} + 2m_d/\tau_{ds})}\mathbf{v_{av}} \\ & + \frac{3[2Y/\Theta - m_u m_d(n_d - n_u)/(\tau_{us}\tau_{ds})]}{e^2 n_u n_d(m_u/\tau_{us} + 2m_d/\tau_{ds})n_u n_d}\mathbf{J} \\ & + \frac{6Z/\Theta - [m_e/\tau_{es} - (m_u/\tau_{us} + 2m_d/\tau_{ds})m_e n_e/\tau_{es}]}{en_u n_d(m_u/\tau_{us} + 2m_d/\tau_{ds})}\mathbf{v_e}\end{aligned} \tag{33}$$

. where,

$$\Theta = (1-x)n_u + n_d + n_s + x n_e \tag{34}$$

$$X = \frac{(n_u - n_d)(n_d - 2n_u)m_u}{\tau_{ud}} - \frac{n_d(n_u - n_e)m_u}{\tau_{ue}} - \frac{2m_d(n_d - n_e)}{\tau_{de}} \tag{35}$$

$$Y = m_u\left(\frac{m_d n_d}{\tau_{ds}} + \frac{m_u n_u}{\tau_{us}}\right)\frac{(n_u - n_d)}{\tau_{ud}} - \frac{m_u m_d n_d(n_u - n_e)}{\tau_{ds}\tau_{ue}} - \frac{m_u m_d(n_d - n_e)}{\tau_{us}\tau_{de}} \tag{36}$$

$$\begin{aligned}Z = {} & \frac{m_u n_e(n_u - n_d)}{\tau_{ud}}\left(\frac{3m_d n_d}{\tau_{ds}} - \frac{m_e^*(n_d - 2n_u)}{\tau_{es}} + \frac{3m_u n_u}{\tau_{us}}\right) \\ & + \frac{m_u n_u n_d(n_u - n_e)}{\tau_{ue}}\left(\frac{m_u}{\tau_{us}} + \frac{2m_d}{\tau_{ds}}\right) - \left(\frac{2m_e^*}{\tau_{es}} + \frac{3m_u}{\tau_{us}}\right)\frac{m_d n_e(n_d - n_e)}{\tau_{de}} \\ & - \left(\frac{m_u}{\tau_{us}} + \frac{2m_d}{\tau_{ds}}\right)\frac{m_d n_d}{\tau_{de}}(n_d - n_e)\end{aligned} \tag{37}$$

The evolution of magnetic field is related to the electric field $\mathbf{E}$ by Faraday's law of induction given by (Jackson 1975)

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E} \tag{38}$$

By a suitable combination of the above relations, one can obtain the governing equation for the magnetic field,

$$\begin{aligned}\frac{\partial \mathbf{B}}{\partial t} &= -\frac{2\Sigma_{i\neq j,s}\frac{m_i n_i}{\tau_{is}}(n_d - n_u)}{3n_u n_d(m_u/\tau_{us} + 2m_d/\tau_{ds})}\nabla \times (\mathbf{v}_{\mathrm{av}} \times \mathbf{B}) \\ &-\left[\frac{3}{e\Theta} + \frac{m_u n_u/\tau_{us} + 2m_d n_d/\tau_{ds}}{e n_u n_d(m_u/\tau_{us} + 2m_d/\tau_{ds})}\right]\nabla \times (\mathbf{J} \times \mathbf{B}) \\ &-\frac{n_e[3m_u n_u/\tau us + 6m_d n_d/\tau_{ds} + 2m_e(n_u - n_d)/\tau es] - 3n_u n_d}{3n_u n_d(m_u/\tau_{us} + 2m_d/\tau_{ds})}\nabla \times (\mathbf{v_e} \times \mathbf{B}) \\ &-\frac{c\Sigma_{i\neq s}m_i n_i/\tau_{is}[6X/\Theta - (m_u n_d/\tau_{us} + 2m_d n_u/\tau_{ds})]}{e n_u n_d(m_u/\tau_{us} + 2m_d/\tau_{ds})}\nabla \times \mathbf{v_{av}} \\ &-\frac{3c[2Y/\Theta - m_u m_d(n_d - n_u)/(\tau_{us}\tau_{ds})]}{e^2 n_u n_d(m_u/\tau_{us} + 2m_d/\tau_{ds})n_u n_d}(\nabla \times \mathbf{J}) \\ &-c\frac{6Z/\Theta - [m_e/\tau_{es} - (m_u/\tau_{us} + 2m_d/\tau_{ds})m_e n_e/\tau_{es}]}{e n_u n_d(m_u/\tau_{us} + 2m_d/\tau_{ds})}(\nabla \times \mathbf{v_e})\end{aligned} \tag{39}$$

. where the electrical conductivity and the other parameters are given by,

$$\sigma_0 = \frac{n_u n_d e^2(\frac{m_u}{\tau_{us}} + \frac{2m_d}{\tau_{ds}})}{3[\frac{2Y}{\Theta} - \frac{m_u m_d(n_d - n_u)}{\tau_{us}\tau_{ds}}]} \tag{40}$$

$$\tag{41}$$

Here in the induction equation we see the appearance of the Ohmic decay, Ambipolar diffusion and Hall drift terms respectively.

### 4.1 Time-scales of field evolution

From the form of induction equation appropriate for the u-d-s-e plasma, the time scale for the ohmic decay is

$$t_{ohmic} = \frac{4\pi\sigma_0 L^2}{c^2} \tag{42}$$

where L is the dimension of the system. Evidently, this is proportional to the length square and is independent of the field strength. We can resolve the ambipolar drift velocity $\mathbf{v_{av}}$ and $\mathbf{f_B}$ into a solenoidal and irrotational component as,

$$\mathbf{v^s_{av}} = \frac{\mathbf{f^s_B} - \frac{6\mathbf{YJ^s}}{e n_u n_d(m_u/\tau_{us}+2m_d/\tau_{ds})} - \frac{2\mathbf{Zv^s_e}}{n_u n_d(m_u/\tau_{us}+2m_d/\tau_{ds})}}{\Sigma_{i\neq s}m_i n_i/\tau_{is}(1 + \frac{2X}{n_u n_d(m_u/\tau_{us}+2m_d/\tau_{ds})})} \tag{43}$$

$$\mathbf{v^{ir}_{av}} = \frac{\mathbf{f}^{ir}_{\mathbf{B}} - \nabla(\Delta\mu) - \frac{6\mathbf{YJ^{ir}}}{e n_u n_d(m_u/\tau_{us}+2m_d/\tau_{ds})} - \frac{2\mathbf{Zv^{ir}_e}}{n_u n_d(m_u/\tau_{us}+2m_d/\tau_{ds})}}{\Sigma_{i\neq s}m_i n_i/\tau_{is}(1 + \frac{2X}{n_u n_d(m_u/\tau_{us}+2m_d/\tau_{ds})})} \tag{44}$$

Therefore, the two timescales corresponding to the solenoidal and the irrotational component of the ambipolar drift velocity are given by :

$$t^s_{ambip} = \frac{4\pi L^2\Sigma_{i\neq s}\frac{m_i n_i}{\tau_{is}}\left(1 + \frac{2\mathbf{X}}{n_u n_d(m_u/\tau_{us}+2m_d/\tau_{ds})}\right)}{B^2 - \frac{24\pi L\mathbf{YJ}^s}{e n_u n_d(m_u/\tau_{us}+2m_d/\tau_{ds})} - \frac{8\pi L\mathbf{Zv}^s_e}{e n_u n_d(m_u/\tau_{us}+2m_d/\tau_{ds})}} \tag{45}$$

$$t^{ir}_{ambip} = \frac{L\Sigma_{i\neq s}\frac{m_i n_i}{\tau_{is}}(1 + \frac{2\mathbf{X}}{n_u n_d(m_u/\tau_{us}+2m_d/\tau_{ds})})}{\mathbf{f^{ir}_B} - \nabla(\Delta\mu) - \frac{6\mathbf{YJ^{ir}}}{e n_u n_d(m_u/\tau_{us}+2m_d/\tau_{ds})} - \frac{2\mathbf{Zv^{ir}_e}}{n_u n_d(m_u/\tau_{us}+2m_d/\tau_{ds})}} \tag{46}$$

The noteworthy aspect of this is that the time-scale associated with the solenoidal drift is a strong function of the field strength and therefore would be very important for strong fields. On the other hand, the irrotational drift is strongly influenced by the chemical imbalance of the plasma. The Hall term, as such, does not directly dissipate the magnetic field. However, this process enhances the cascading of energy from large-scale structures to small-scale ones. In other words, lower multi-poles of the magnetic field of the magnetic field(like dipole) would convert into higher multi-poles. Since, higher multi-poles would dissipate faster through the ohmic channel, the Hall term is effective in fast dissipation of the magnetic field indirectly.

**Comments** - The next part of the work is to make numerical estimates of the above time-scales in order to compare them with the observed astrophysical results. This work is being continued to obtain such numerical results in future (next semester). We are also looking at the more general case of the strange star when the $u - d - s$ plasma also contains a small admixture of electrons.

## References


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