This expression has to be modified if the material is a thin wafer whose thickness is comparable to the probe-spacing. For a large, flat, wafer, of thickness t, one has the formula :

$$\sigma = \frac{I}{\pi t V \ln 2}.$$

For bulk samples a simple arrangement can be used. This of course needs special mechanical cutting. The current leads can be soldered at the ends and the voltage leads to two electrodes properly cut and formed.

For measurement at different temperatures this system can be placed in a furnace in good thermal contact with a thermocouple for measuring the temperature.

Hall Effect in Semi Conductors :

As we have defined earlier, the Hall coefficient R of a conductor with one type of charged carrier is given by :

$$R = \frac{E_y}{j_x H_z} = -\frac{1}{nec} \tag{49}$$

For n-type semiconductors : $R_h = -\dfrac{1}{nec}$

while for p-type material, $R_e = \dfrac{1}{pec}$.

Hence a measurement of R should reveal the sign of the charged carrier and also provide a measure of its concentration n or p. Equation (49) is however derived under the assumption that the relaxation times for the electrons and the holes τ_e and τ_h are independent of the velocity of the carriers.

When two types of carriers (electrons and holes) are present simultaneously in the material, the expression for Hall coefficient becomes more complicated and is given by :

$$R = -\frac{1}{ec} \cdot \frac{n\mu_e^2 - p\mu_h^2}{(n\mu_e + p\mu_h)^2} \tag{50}$$

where n, p are the carrier concentrations and μ_e, μ_h are their mobilities. This expression is derived as follows :

We consider a rectangular slab of material containing only two types of carriers, electrons and holes, placed in a uniform magnetic field H_z in the z-direction. Let the current density in the x-direction be j_x under the influence of an external electric field E_x. Then we have :

$$j_x = e(n\mu_e + p\mu_h)\, E_x \tag{51}$$

where the average drift velocities in the x-direction are :

$$v_e = -\mu_e E_x. \text{ and } v_h = \mu_h E_x$$

in the absence of the magnetic field.

Now when the magnetic field H_z is applied in the z-direction there will be a transverse Lorentz force :

$$F_y = \frac{q}{c}(v \times H)_y$$

on both the electrons and the holes. For the electrons :

$$F_y = -\frac{e}{c} v_e H_z$$

and the deflection due to the Lorentz force will be in the positive y-direction (the electrons drift in the negative x-direction when the conventional current j_x is in the positive x-direction). For the holes, the drift velocity is in the positive x-direction and since their charge is +e, they will also be deflected in the positive y-direction i.e. in the same direction as the electrons. This can be seen by the application of the left-hand rule. An equilibrium will be established when the net current in the y-direction vanishes ; that is, charges are set up so that the electric field E_y generated along the y-axis produces forces on both the electrons and the holes which balance the Lorentz forces on them. However, since the electrons and holes travel with different velocities, then the force due to the transverse electric field E_y only balances on the average and there is a tendency of the electrons and holes to be deflected to one side separately. This produces a small net y-component of current due to each type of carrier which tends to change the current densities in the x direction slightly. If we define the Hall coefficient :

$$R = E_y / j_x H_z,$$

we have to calculate this changed current density j_x because of the Hall field generated in the x-direction because of the currents j_e and j_h in the y-direction for each type of carrier. The net current : $j_y = j_e + j_h = 0$ in equilibrium. We write R_e and R_h for the Hall Coefficients corresponding to electron and hole motion respectively.

Let us write :

$$\sigma = e\,(n\mu_e + p\mu_h) = \sigma_e + \sigma_h \tag{52}$$

where $\sigma_e = en\mu_e$, $\sigma_h = ep\mu_h$ (53)

The electric field in the x-direction generated due to electron current j_{ye} in the y-direction is $R_e j_{ye} H_z$ and is in the positive x-direction by the left hand rule. Then the net electron current in the x-direction, j_{xe}, is given by (writing $H = H_z$) :

$$E_x + R_e j_{ye} H = j_{xe}/\sigma_e \qquad (54)$$

or, $$j_{xe} - R_e H \sigma_e j_{ye} = \sigma_e E_x \qquad (55)$$

The electric field E_y in the y-direction is due to the current in the y-direction and the Hall-field. Then :

$$E_y - R_e H j_{xe} = j_{ye}/\sigma_e$$

or $$R_e H \sigma_e j_{xe} + j_{ye} = \sigma E_y \qquad (56)$$

the material being assumed isotropic so that the conductivity σ_e is the same in all directions. Solving for j_{xe} and j_{ye} in (55) and (56) we get :

$$j_{xe} = \frac{\sigma_e E_x + R_e H \sigma_e^2 E_y}{1 + R_e^2 H^2 \sigma_e^2} \qquad (57)$$

$$j_{ye} = \frac{\sigma_e E_y - R_e H \sigma_e^2 E_x}{1 + R_e^2 H^2 \sigma_e^2} \qquad (58)$$

Similarly, the hole currents are given by :

$$j_{xh} = \frac{\sigma_h E_x + R_h H \sigma_h^2 E_y}{1 + R^2 H^2 \sigma_h^2} \qquad (59)$$

$$j_{yh} = \frac{\sigma_h E_y - R_h H \sigma_h^2 E_x}{1 + R_h^2 H^2 \sigma_h^2} \qquad (60)$$

By adding the expressions (58) and (60) one can calculate j_y and equate it to zero so as to get the equilibrium conditions. Since the magnetic field used in Hall effect measurements is only of moderate intensity (2000-6000 oersted), one can neglect the term in H^2 in the denominator. Then from (58) and (60) :

$$j_y = j_{ye} + j_{yh} \approx E_y(\sigma_e + \sigma_h) - (R_e \sigma_e^2 + R_h \sigma_h^2)\, H E_x = 0 \qquad (61)$$

also we have

$$E_x = \frac{j_x}{\sigma_e + \sigma_h} \qquad (62)$$

Then substituting (62) in (61), the Hall coefficient of the semiconductor becomes :

$$R = \frac{E_y}{j_x H} = \frac{R_e \sigma_e^2 + R_h \sigma_h^2}{(\sigma_e + \sigma_h)^2} \qquad (63)$$

Now $R_e = -\frac{1}{nec}$ and $R_h = \frac{1}{pec}$

and substituting for σ_e and σ_h from (53) we get.

$$R = -\frac{1}{ec} \frac{n\mu_e^2 - p\mu_h^2}{(n\mu_e + p\mu_h)^2}. \qquad (64)$$

Importance of Hall Effect Measurements :

The Hall effect gives a direct proof that there are positively charged carriers (holes) in a p-type semiconductor. Secondly, in combination with measurements of electrical conductivity, it gives a value for the mobility of the majority carriers. For example, for n-type semiconductors, the electrical conductivity : $\sigma = n\mu_e$ and the Hall coefficient $R_e = -\frac{1}{nec}$. so that $\sigma R_e = \frac{-\mu_e}{e_c}$. The product $R\sigma$ is known as the *Hall Mobility* and usually denoted by μ_H.

Unfortunately, the interpretation of Hall effect measurements in semiconductors is rather complicated because of the complex band structure of semiconductors and as a result the simple formula for R is not always valid. Further it is found that the Hall coefficient R in semiconductors is dependent on the applied magnetic field H unless the field is very low or very high when R is found to be linear in H.

The relaxation time of the carriers depends on the mechanism of scattering and is usually expressed as a function of energy through an empirical relation of the form $\tau = \text{const. } E^p$. With $p = \frac{1}{2}$, one gets on averaging over the velocity distribution of the carriers :

$$R = \frac{1}{nec} \frac{\langle v^2\tau\rangle \langle v^2\rangle}{\langle v^2\tau\rangle^2} = \frac{3\pi}{8} \cdot \frac{1}{nec}.$$

Life Time and Recombination of electrons and holes :

At moderate temperatures, electrons in the conduction band and holes in the valence band in an intrinsic semiconductor are produced continuously by thermal excitation. The excited electrons do not remain indefinitely in the conduction band but continuously make direct or indirect transitions to the valence band and recombine with the holes. The process is known as *recombination* of electrons and holes. The indirect transitions to the valence band take place via trapping centres or *traps* which exist within the crystal. This mechanism is predominant in Ge and Si. In equilibrium, the rate of gene-