Sem-I - Newtonian Mechanics

(Instructor: AKB, Department of Physics, Asutosh College)

Assignment III: Central Force, Gravitation, Collisions

Submission due date: 18/02/2026

Q.1) **(a)** A system of particles with masses m_i and position vectors $\bar{r}_i (i = 1, 2, \cdots, n)$ moves under its own mutual gravitational attraction alone. Write down the equation of motion $\bar{r}_i$. Show that a possible solution of the equation of motion is given by $\bar{r}_i = t^{2/3}\bar{a}_i$, where $\bar{a}_i$'s are constant vectors satisfying

$$\bar{a}_i = \frac{9G}{2}\sum_{j\neq i}^{n} \frac{m_j(\bar{a}_i - \bar{a}_j)}{\left[\bar{a}_i - \bar{a}_j\right]^3},$$

where G is the Gravitational constant. Show that for this system, the total angular momentum about the origin and the total linear momentum both vanish. What is the angular momentum about any other fixed point?

(b) Find the central force for which the orbit is given by $r = ke^{a\theta}$, where a and k are constants.

(c) A particle of mass m moves under the action of a central force whose potential is $V(r) = kr^4, K > 0$. For what energy and angular momentum will the orbit be a circle of radius a about the origin? What will be the period? If the particle is slightly disturbed from this circular motion, what will be the period of small radial oscillations about $r = a$?

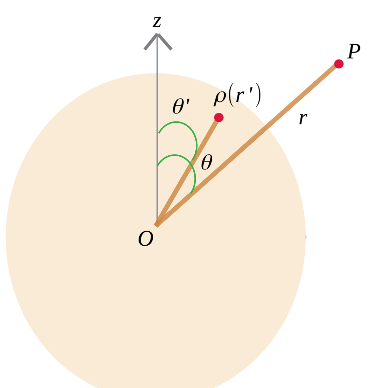
(d) The attractive nuclear force between a neutron and proton is given by the *Yukawa potential* $V(r) = \frac{Ke^{-\alpha r}}{r}, K > 0$. Find and compare the force with inverse square law and discuss the types of motion that differs the inverse square law. Find angular momentum, total energy, the period of circular motion and the period of small radial oscillations.

Q.2) **(a)** If the density of the material within a spherical body varies inversely as the distance from the centre, show that the gravitational field inside is the same everywhere.

(b) Consider a pair of stars of equal mass M rotating about their common centre of mass. The attraction between the stars is gravitational and the stars keep a separation l between them. Show that the time period of rotation of this double star system is given by $\pi l\sqrt{\frac{2l}{GM}}$, where G is the universal Gravitational constant.

(c) A planet of mass m moves around the Sun of mass M. The nearest and the farthest distance of the planet from the Sun are a and b respectively. Find the magnitude of the angular momentum of the planet around the Sun in terms of m, M, a, b and G, where G is the gravitational constant.

Q.3)



(a) Consider an axially symmetric and bounded arbitrary mass M of density $\rho(\mathbf{r})$ as displayed beside. The gravitational potential at a point P, that lies at a large distance r, is of the form

$$V(r,\theta) \approx -\frac{GM}{r} + \frac{f(\theta)}{r^2} + \cdots,$$

where $M = 2\pi \int \rho(r', \theta') r'^2 \sin\theta' dr' d\theta'$. Find the form of $f(\theta)$.

(b) A test mass of density $\phi(\mathbf{r})$ is placed in the gravitational potential $V(\mathbf{r})$. What will be its gravitational potential energy?

Q.4) (a) Consider a head-on elastic collision in one dimension between a heavy moving mass m_1 and a light mass m_2 at rest ($m_2 \gg m_1$). Show that after collision, the light mass rebounds with a speed equal to twice the initial speed of m_1.

(b) A ball moving with speed 9m/s strikes an identical stationary ball such that after collision, the direction of each ball makes an angle 30° with the original line of motion. Find the speeds of two balls after collision. Is the kinetic energy conserved in this collision?