## Sem-I - General Properties of Matter

**(Instructor: AKB, Department of Physics, Asutosh College)**

**Assignment II: Hydrostatics & Hydrodynamics**

**Submission due date: 09/01/2022**

**Q.1)** Water stands at a depth $h$ in a large open drum whose side-walls are vertical. A hole is made in one of the walls at a depth $x$ below the water surface. **(i)** Calculate the horizontal distance $d$ from the wall at which the emerging stream of water strikes the ground. **(ii)** What should be the value of $x$ which makes $d$ maximum?

**Q.2)** Two drums $A$ and $B$ contain different liquids and both have a small hole on the vertical wall at a depth $h$ below the liquid surface. The hole in drum $A$ has half the cross-sectional area of the hole in drum $B$. **(i)** Using Toricelli's theorem, Calculate the ratio $\rho_A/\rho_B$ of the densities of the liquids, if the mass flow rate is the same for the two holes. **(ii)** What is the ratio $V_A/V_B$ of the volume flow rates from the two drums? **(iii)** At one instant, the liquid in drum $A$ is $12.0cm$ above the hole. If the drums are to have equal volume flow rates, what height above the hole must the liquid in drum $B$ be just then?

**Q.3)** Two streams merge to form a river. One stream has a width of $8.2m$, depth of $3.4m$, and current speed of $2.3m/s$. The other stream is $6.8m$ wide and $3.2m$ deep and flows at $2.6m/s$. If the river has width of $10.5m$ and speed at $2.9m/s$, what is its depth?

**Q.4)** Dead Sea is about seven times saltier than any ocean due to the high evaporation rate and low rainfall. About one-third of the body of a swimmer floating in the Dead Sea is above the waterline. Assuming that the humanbody density is $0.98gm/cm^3$, find the density of the water in the Dead Sea.

**Q.5)** Water flows along a horizontal tube of which the cross-section is not constant. Using Bernoulli's theorem, calculate the change in pressure when the velocity of flow changes from $10cm/s$ to $20cm/s$.

**Q.6)** A hypothetical material out of which an astronomical object is formed has an equation of state

$$p = \frac{1}{2}K\rho^2,$$

where $p$ is the pressure and $\rho$ is the mass density. Show that for this material under conditions of hydrostatic equilibrium, there is a linear relation between the density and the gravitational potential.

**Q.7)** The vorticity $\boldsymbol{\omega}$ of a flow field is defined as curl of the velocity vector $\boldsymbol{\omega} = \boldsymbol{\nabla} \times \mathbf{v}$. Show that for an incompressible fluid, the following relation holds between the velocity and vorticity vector fields,

$$\boldsymbol{\nabla} \cdot [(\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{v}] = \frac{1}{2}\nabla^2(\mathbf{v} \cdot \mathbf{v}) - \mathbf{v} \cdot \nabla^2\mathbf{v} - \boldsymbol{\omega} \cdot \boldsymbol{\omega}.$$

**Q.8)** An incompressible fluid of mass density $\rho$, viscosity $\eta$ is pumped in steady-state laminar flow through a circular pipe of internal radius $R$ and length $L$. The pressure at the inlet end is $p_1$ and the pressure at the exit is $p_2$ with $p_1 > p_2$. Let $Q$ be the mass of fluid that flows through the pipe per unit time. Using steady state limit of the Euler's equation for viscous liquid (or Navier-Stokes equation for "wet water" as in Feynman Lectures)

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{v}\right] = -\boldsymbol{\nabla} p + \eta\nabla^2\mathbf{v} + \mathbf{F}_{\text{body}},$$

determine $Q$, which is the Poiseuille's equation. *[Hint: Switch to cylindrical coordinates.]*