2024

PHYSICS — HONOURS

Paper : DSCC-1

(Basic Physics - I)

Full Marks : 75

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.* 1** and ***any five*** questions, taking ***at least one*** from ***each Group***.

1. Answer ***any five*** questions : 3×5

(a) Sketch the functions $f(x)=|x|$ and $g(x)=\frac{df}{dx}$, in the range $-2 \le x \le +2$. Comment on the continuity and differentiability of the given functions.

(b) Construct a differential equation of the form $y''(x)+ay'(x)+by(x)=0$, where '′' indicates derivative with respect to x and a, b are constants, for which the solutions are e^{2x} and e^{-2x}.

(c) Evaluate the integral $\iint\limits_S \sqrt{x^2+y^2}\,dxdy$ (by transforming to plane polar coordinates or otherwise), where S is a region in the x-y plane bounded by $x^2+y^2=4$ and $x^2+y^2=9$.

(d) Find the first three non-zero coefficients in the Taylor series expansion of $\frac{1}{\cos x}$ about $x=0$.

(e) A particle is moving in 3-dimensions under a conservative force field. Show that the sum of its kinetic and potential energies is conserved.

(f) A mass m with speed v approaches a stationary mass M. The masses bounces off each other. Find the initial velocities of the particles in the CM frame. Assume that the motion takes place in 1-dimension.

(g) A system consists of two particles of masses m_1 and m_2. Prove that the centre of mass divides the line joining m_1 and m_2 into two segments whose lengths are in the ratio $m_2 : m_1$.

(h) What are streamlines? Explain why two streamlines can never intersect.

Group - A

2. (a) If $z=\ln\sqrt{x^2+y^2}$, show that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=1$.

(b) Show that $df=y(1+x-x^2)dx+x(x+1)dy$ is not an exact differential. Find out a suitable integrating factor so that the differential becomes an exact differential.

Please Turn Over

(c) Plot $r = a(1 - \cos\theta)$ in polar coordinates (r, θ) in the range $0 \le \theta \le 2\pi$, where a is a constant.

(d) Solve the differential equation $x''(t) - x(t) = 0$, with the boundary conditions $x(0) = 1$ and $x(\infty) = 0$. Here '$'$' indicates derivative with respect to t. 3+4+2+3

3. (a) Using the summation convention and Levi-Civita symbol, show that $\vec{\nabla} \times \vec{\nabla}\phi = 0$, where $\phi(x, y, z)$ is any scalar field.

(b) Find the flux of the vector $\vec{F} = z\hat{K}$ over a sphere of radius a centered at origin.

(c) For a scalar field $\Phi(\vec{r})$, show that $\vec{\nabla}\Phi(\vec{r})$ is always normal to the Φ = constant surface at $\vec{r}$. Find a unit vector normal to the surface $x^2 + y^2 + z^2 = 3$ at (1,1,1).

(d) Given $\vec{F} = z^2\hat{i} + xy\hat{j} + x^2\hat{k}$, find $\nabla^2\vec{F}$. 3+3+(2+2)+2

4. (a) Find the line integral of the vector $\vec{F} = x^2\hat{i} + y^2\hat{j} + (xz - y)\hat{k}$ along the straight line joining the points (0, 0, 0) to (2, 3, 4).

(b) Solve the differential equation $y'' - 5y' + 6y = 2\cos x$.

(c) Find the expression for $\vec{\nabla}$ in cylindrical coordinate system. 3+5+4

Group - B

5. (a) A particle of mass m, moving in a horizontal line with initial velocity v_0 is acted upon by a resistive force γv^2 (γ is constant).

(i) Set up the differential equation of motion of the particle.

(ii) Find the speed $v(t)$.

(iii) Plot $v(t)$ against t.

(b) The coordinates of a particle moving in the x-y plane are given by $x(t) = at$ and $y(t) = bt^2$, where $a = 2$ m/s and $b = 0.5$ m/s^2. Find the velocity and acceleration of the particle at $t = 3$ Sec. Find the equation of the trajectory of the particle. (1+3+2)+(4+2)

6. (a) A particle of mass m and total energy E moves along the x-axis in a potential $V(x)$, starting from x_0 at time t_0. If the particle reaches x at time t, show that

$$t - t_0 = \pm \int_{x_0}^{x} dx' \frac{1}{\sqrt{\frac{2}{m}[E - V(x')]}}.$$

(b) A particle of mass m moves in a potential field $V(x) = m(x^3 - 3x)$.

(i) Sketch the potential $V(x)$ against x.

(ii) Find the positions of stable and unstable equilibrium.

(iii) Find the force acting on the particle.

(iv) If the particle is displaced slightly from stable equilibrium, then find the frequency of small oscillation. 4+(2+3+1+2)

7. (a) Show that if the total linear momentum of a system of particles is zero, the angular momentum of the system is independent of the choice of origin.

(b) Show that for a system of particles $\frac{d\vec{L}}{dt} = \vec{N}$, where $\vec{L}$ is the total angular momentum and $\vec{N}$ is the total external torque (assume internal forces are pairwise central forces).

(c) Find out the gravitational potential at a point inside a uniform solid sphere. 3+5+4

8. (a) For a particle of mass m moving in a central potential $U(r)$, show the energy is given in polar coordinates by $E = \frac{1}{2}m\dot{r}^2 + U_{eff}(r)$, where $U_{eff}(r) = U(r) + \frac{\ell^2}{2mr^2}$, ℓ being the angular momentum.

(b) A particle of mass m moving in a central force field describes a spiral $r = k\theta^2$, where k is a positive constant.

(i) Find the force law.

(ii) Find the total energy of the system.

(c) Consider a head-on elastic collision in one dimension between a heavy mass (m_1) and a light mass (m_2) ($m_1 >> m_2$), where m_2 is initially at rest. Show that after the collision, the light mass rebounds with a speed equal to about twice the initial speed of m_1. 3+(2½+2½)+4

9. (a) Write down the equation of continuity for a moving fluid. Explain the physical significance of this equation.

(b) Establish that the condition of equilibrium of an incompressible fluid under body force $\vec{F}$ per unit mass is given as $\vec{F} = \frac{1}{\rho}\vec{\nabla}P$ when P is the pressure at some point (x, y, z) and ρ is the density of the fluid. Hence establish Pascal's law.

(c) Using Bernoulli's theorem for an incompressible fluid, show with suitable example that high velocity point on a streamline corresponds to low pressure and vice versa. 3+(4+2)+3

2023

PHYSICS — HONOURS

Paper : DSCC-1

(Basic Physics-I)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any five*** questions, taking at least ***one*** question from ***each Group.***

1. Answer ***any five*** questions : 3×5

(a) Sketch schematically the function $f(x) = |x|^{\beta}$ (where β is a parameter) in a single plot for three different values of β = ½, 1, 2, in the range $-1 \le x \le +1$. For which of the above values of β is the function differentiable at the origin?

(b) If $\vec{a} \times \vec{g} = \vec{b}$ and $\vec{a} \cdot \vec{g} = \Phi$, express $\vec{g}$ in terms of $\vec{a}, \vec{b}, \Phi$ and $|\vec{a}|$.

(c) Expand $\dfrac{1}{x-2}$ in a Taylor series about the point $x = 1$.

(d) Solve the differential equation $\dfrac{dy}{dx} + 2y = x^2$.

(e) A particle moves in 2-dimensions on the ellipse : $x^2 + 4y^2 = 1$. At a particular instant, it is at the point $(x, y) = (2, 1)$ and the x-component of its velocity is 2 (in suitable units). Find the y-component of its velocity at that instant.

(f) Show that the centre of mass of a system of particles is unique.

(g) Show that the areal velocity of a particle moving in x-y plane under a central force is constant.

(h) A horizontally placed hollow tube has a cross-sectional area A at the beginning of the tube that gradually tapers off to $\frac{A}{2}$ at the end. An incompressible, ideal fluid of density ρ enters the tube with a velocity v at the beginning of the tube. What is the difference in pressure at the two ends of the tube?

Group - A

2. (a) Evaluate $\lim\limits_{x \to 0} \dfrac{1-\cos^n x}{x^2}$.

(b) Consider a function $f(x) = Ae^{-\lambda x}$ with the constraint $\int_{-\infty}^{\infty} f^2(x)dx = 1$. x has the dimension of length. Using dimensional analysis, find dimension of A in terms of λ.

(c) Examine whether the differential equation $x(x^2 + 2y^2)\, dx + y\,(2x^2 + y^2)\, dy = 0$ is exact. Solve the equation, if exact.

(d) Given that $x''(t) - 4x'(t) + 4x(t) = 0$ with $x(0) = 1$ and $x'(0) = 0$, find out $x\left(-\frac{1}{2}\right)$, where $'$ indicates derivative with respect to t.

2+3+(1+2)+4

3. (a) Using the summation convention and Levi-Civita symbol, show that $\vec{A}\cdot\left(\vec{A}\times\vec{B}\right) = 0$.

(b) If $\vec{A}$ is an irrotational vector field, show that $\vec{A}\times\vec{r}$ is solenoidal.

(c) Given that $\Phi\,(x, y, z) = xy + \sin z$, find $\vec{\nabla}\Phi$ at $\left(1, 2, \frac{1}{2}\pi\right)$. How fast is Φ increasing in the direction of $4\hat{i} + 3\hat{j}$ at $\left(1, 2, \frac{1}{2}\pi\right)$?

(d) Prove that $\oiint_S d\vec{S} = 0$ for any closed surface S.

3+3+(2+1)+3

4. (a) Calculate $\iint_S \vec{\nabla}\times\vec{A}\cdot d\vec{S}$, where $\vec{A} = -y\hat{i} + x\hat{j}$ and S is the open surface area of hemisphere $x^2 + y^2 + z^2 = a^2,\ z \geq 0$.

(b) Find the expression of $\dfrac{\partial}{\partial x}$ in terms of plane polar coordinates.

(c) Suppose a particle is moving along a trajectory given by $r = ct$, $\theta = \Omega\, t$, where c and Ω are positive constants. Find the velocity and acceleration vectors of the particle at time t. 4+4+(2+2)

Group - B

5. (a) A particle of mass m is falling under gravity in presence of a drag force $-\gamma v$, where v is the speed and $\gamma(>0)$ is a constant.

(i) Set up the equation of motion of the particle.

(ii) Find the speed $v(t)$ at a later time t assuming $v(t = 0) = 0$.

(iii) Show that $v\left(t \to \infty\right) = \dfrac{mg}{\gamma}$.

(iv) Plot $v(t)$ against t.

(v) From the expression of $v(t)$ obtained, recover the familiar expression of speed in the drag-free case.

(b) A particle is moving on a circular path with a constant speed u. Find the magnitude of the change in its velocity after it has swept an angle θ.

(1+3+1+2+2)+3

6. (a) A particle moves in 1-dimension in a potential $V(x)$ having a local minima at x_0. Show that for small displacements around x_0, the particle behaves like a harmonic oscillator.

(b) A particle of mass m is moving in the region $x > 0$ under the influence of the potential $U(x) = U_0\left(\frac{\alpha}{x} + \frac{x}{\alpha}\right)$, where U_0 and α are positive constants. (i) Find the force acting on the particle. (ii) Find the positions of equilibrium and identify the stable one. (iii) Find the frequency of small oscillation about this point.

(c) Show that for a particle moving in a conservative force field $\vec{F}$, the integral $\int_1^2 \vec{F}\cdot d\vec{r} = T_2 - T_1$, where T is the kinetic energy of the particle.

4+(1+2+1)+4

7. (a) The position vector of a moving particle at any instant of time t is given by $\vec{r} = \left(2+3t^2\right)\hat{i} + 5t^2\hat{j} + t\hat{k}$. Find the force $\vec{F}$, torque $\vec{N}$ and angular momentum $\vec{L}$ of the particle about the origin. Hence, verify that $\vec{N} = \frac{d\vec{L}}{dt}$.

(b) For a system of particles, show that the angular momentum about a point is equal to the angular momentum of a single particle of total mass $M\left(=\sum_i m_i\right)$ situated at the centre of mass together with the angular momentum of the system of particles about the centre of mass.

(1+2+2+1)+6

8. (a) A particle is thrown from the Earth's surface with speed $v = \sqrt{\frac{3GM}{2R}}$, where M and R are mass and radius of the Earth respectively. What will be the nature of the orbit of the particle?

(b) Find the central force for which the orbit is given by $r = ke^{a\theta}$, where a and k are constants.

(c) A planet of mass m moves around the Sun of mass M. The nearest and the farthest distance of the planet from the Sun are a and b respectively. Find the magnitude of the angular momentum of the planet around the Sun in terms of m, M, a, b and G, where G is the gravitational constant.

(d) A ball moving with speed of 9 m/s strikes an identical stationary ball such that after the collision, the direction of each ball makes an angle 30° with the original line of motion. Find the speeds of two balls after collision. Is the kinetic energy conserved in this collision? 2+3+3+(2+2)

9. (a) Set up the equation of continuity expressing local conservation of mass, in the context of fluid motion.

(b) Explain Newtonian liquid and non-Newtonian liquid with one example in each case.

(c) (i) Establish Euler's equation for an ideal fluid moving in the presence of gravity.

(ii) Hence, deduce that in a fluid at rest, the pressure difference between two points inside the fluid, separated by a vertical height h is $h\rho g$, where ρ is the density of the fluid and g is acceleration due to gravity.

3+(2+1)+(4+2)

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2022

PHYSICS — HONOURS

[For Syllabus : 2019-2020 & 2018-2019]

Paper : CC-2

(Mechanics)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** from the rest.

1. Answer ***any five*** questions : 2×5

(a) Show that Newton's laws of motion remain invariant under Galilean transformation.

(b) A particle moves under the influence of a force $\vec{F}$ and has an instantaneous velocity $\vec{V}$. Show that $\frac{dT}{dt} = \vec{F}.\vec{V}$, where T is the kinetic energy of the particle.

(c) A particle has total energy E and the force on it is due to potential field $V(x)$. Show that the time taken by the particle to go from x_1 to x_2 is

$$t_2 - t_1 = \sqrt{\frac{M}{2}} \int_{x_1}^{x_2} \frac{dx}{\sqrt{E - V(x)}}$$

assuming the motion to be one-dimensional and that the particle does not reverse its motion.

(d) What is the rotational period of a binary star consisting of two equal mass M and separated by a distance L? [Treat the binary star as a two-body system.]

(e) Show that the trajectory of a particle moving under a central force is confined in a plane.

(f) Angular momentum and angular velocity of a rigid body are not always parallel. Justify the statement.

(g) In streamline flow of Newtonian fluid two streamlines never intersect.— Explain.

2. (a) If $\hat{r}$ and $\hat{\phi}$ denote the unit vectors in plane polar coordinates then,

(i) Prove that $\hat{r}$ and $\hat{\phi}$ forms an orthogonal co-ordinate system.

(ii) Show that $\dot{\hat{r}} = \dot{\phi}\,\hat{\phi}$ and $\dot{\hat{\phi}} = -\dot{\phi}\,\hat{r}$.

(iii) Express the acceleration $\ddot{\vec{r}}(t)$ in the plane polar co-ordinate system.

(b) Set up the equation of motion for a particle falling under constant gravity with a resistive force that is proportional to its velocity. Find out the velocity as a function of time assuming that the body starts from rest. Comment on the motion after a large time. (2+2+2)+(1+2+1)

3. (a) Show that for conservative force $\vec{F}(\vec{r})$ the relation curl $\vec{F}(\vec{r}) = 0$ holds.

(b) The components of a force in the xy plane are :

$$F_x = x^3 + xy^2$$
$$F_y = y^3 + 3x^2y$$

Compute the work done in a displacement from the point (1, 1) to (2, 2) along the following paths :

(i) a straight line from (1, 1) to (2, 2)

(ii) a straight line from (1, 1) to (1, 2) followed by another straight line from (1, 2) to (2, 2). Could the force field be conservative?

(c) A particle of mass m at rest at (a, 0, 0) is subject to a force $\vec{F} = -\frac{k}{x^3}\hat{i}$ where k is a constant. Find the time taken by the particle to reach the origin. 2+(2+2)+4

4. (a) A frame of reference S' rotates with uniform angular velocity $\vec{\omega}$ with respect to a stationary frame S having common origin. Establish the identity,

$$\frac{d}{dt} = \frac{d'}{dt} + \vec{\omega} \times$$

(b) Hence, obtain expression for centrifugal force and coriolis force for the motion of a particle with respect to the rotating frame.

(c) A particle of mass m is dropped vertically from a height 'h' on north pole. What is the expected deflection of the particle due to coriolis force? 4+4+2

5. (a) Define moment of inertia and principal axis of a rigid body.

(b) Three particles each of mass m are situated at (a, 0, 0,), (0, a, 0) and (0, 0, a). Set up the principal axes of the system and calculate the principal moments of inertia.

(c) A rigid body rotates about an axis having direction cosines (l, m, n) with angular velocity $\vec{\omega}$. Show that the kinetic energy of rotation of the body is, $T = \frac{1}{2} I_{(\mathrm{l,m,n})}\omega^2$, where $I_{\mathrm{l,\,m,\,n}}$ is the moment of inertia of the body about the axis of rotation. 2+4+4

6. (a) Prove that if the centre of mass of a system of particles remains at rest, the total angular momentum of the system of particles about 'any' point is the same and is equal to the angular momentum about the fixed centre of mass.

(b) If the density of the material within a spherical body varies inversely as the distance from the centre, show that the gravitational field inside is the same everywhere. 5+5

7. (a) Set up Euler's equation of hydrodynamics for an incompressible fluid.

(b) Use the relation $\vec{F} = \vec{\nabla} p$ for a fluid at rest (where the symbols have their usual meanings) to establish Archimedes principle.

(c) Water flows through a horizontal tapering tube of circular cross section, the diameter of the entrance and exit ends being 10 cm and 7.5 cm respectively. The pressure difference at two ends is 10 cm of mercury. What is the rate of flow of water through the tube? 4+3+3

T(III)-Physics-H-5

2021

PHYSICS — HONOURS

Fifth Paper

Full Marks : 100

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***any ten*** questions : 2×10

(a) Is the constraint given by $x\dot{x} + y\dot{y} + x\dot{y} + \dot{x}y = k$ (a constant), a holonomic constraint?

(b) Show that the two Lagrangians $L_1 = (q - \dot{q})^2$ and $L_2 = \left(q^2 + \dot{q}^2\right)$ are equivalent.

(c) Prove that for motion of a particle under central force, the areal velocity with respect to the centre of force remains constant.

(d) If the kinetic energy $T = \frac{1}{2} m\, \dot{r}^2$ and the potential energy $V = \frac{1}{r}\left(1 + \frac{r^2}{c^2}\right)$, find the Hamiltonian '$H$' and determine whether $H = T + V$.

(e) Explain what is meant by streamlines.

(f) Derive the equation of continuity for a compressible fluid.

(g) For a four vector A^{μ} show that $A_{\mu} A^{\mu}$ is a scalar.

(h) Find the constant C which makes $e^{-\alpha x^2}$ an eigenstate of the operator. $\frac{d^2}{dx^2} - Ex^2$ (α is a constant).

(i) Can we measure the kinetic and potential energies of a particle simultaneously with arbitrary precision?

(j) Why are the Stokes lines brighter than anti-Stokes lines in Raman Spectra?

(k) The electronic configuration of Mg is $1s^2\ 2s^2\ 2p^6\ 3s^2$. Obtain its spectral term.

(l) Why is pure vibrational spectra observed in liquid?

Please Turn Over

Group - A

Section - I

(Classical Mechanics II)

Answer ***any two*** questions.

2. (a) Starting from Lagrange's equation of motion, obtain Hamilton's equation of motion using Legendre transformation.

(b) For the Hamiltonian $H = q_1p_1 - q_2p_2 - aq_1^2 + bq_2^2$, solve the Hamilton's equation of motion and prove that q_1q_2 = constant and $\frac{(p_2 - bq_2)}{q_1}$ = constant.

(c) Show that the effective potential of a particle of mass 'm' in a central force field is given by $U_{eff}(r) = U(r) + \frac{L^2}{2mr^2}$, where L is the angular momentum. 4+3+3

3. (a) Consider a simple harmonic oscillator with angular frequency ω_0. What will be its angular frequency when a constant force K is applied on it?

(b) The point of suspension of a simple pendulum moves simple harmonically along the vertical line. Obtain the Lagrangian of the system.

(c) Prove that, if the Lagrangian of an unconstrained system is invariant under continuous translation, then the total linear momentum is conserved. 3+4+3

4. (a) State Bernoulli's equation of fluid motion and mention the conditions of its validity.

(b) The Lagrangian of a particle of mass m is $L = \frac{1}{2}\left(m\dot{x}^2 - bx^2\right)e^{at}$ where a and b are positive constants. Determine the Hamiltonian. Is it a constant of motion?

(c) A flat vertical plate is struck normally by a horizontal jet of water 50 mm in diameter with a velocity of 18 m/s. Calculate the force on the plate assuming it to be stationary. 3+4+3

Section - II

(Special Theory of Relativity)

Answer ***any two*** questions.

5. (a) Define the interval between two events in space time. Show that it is invariant under a Lorentz transformation. Hence explain the conditions for which the interval is time-like, space-like or light-like.

(b) A muon at rest has life time 2×10^{-6} sec. What is its life time when it travels with a velocity $\frac{3}{5}$c ?

(c) Define covariant and contravariant vector. (1+2+3)+2+2

6. (a) Discuss about inconsistency, if any, in Newton's law of gravitation in the light of postulates of special theory of relativity.

(b) Define Minkowski space. Show that Lorentz transformation can be regarded as transformation due to a rotation of axes through an imaginary angle given by $\theta = tan^{-1}(i\beta)$ where $\beta = \frac{v}{c}$ in the 4-dimensional Minkowski space.

(c) Two rods of proper length l_0 move lengthwise towards each other parallel to the common axis with the same velocity v relative to the laboratory frame. Show that the length of each rod in the reference frame fixed to the other rod is $l = l_0 \frac{(1-\beta^2)}{(1+\beta^2)}$, $\beta = \frac{v}{c}$. 2+(1+3)+4

7. (a) Define proper time interval $d\tau$. Hence construct velocity four vector. Show that it is a time-like vector.

(b) If $A^{\mu\nu}$ and $B^{\mu\nu}$ are two tensors, Show that $A^{\mu\nu} B_{\mu\nu} = A_{\mu\nu} B^{\mu\nu}$.

(c) For two four vectors A and B, prove that $A_\mu B^\mu = A^\mu B_\mu$. 4+4+2

Group - B
Section - I
(Quantum Mechanics II)

Answer ***any two*** questions.

8. (a) Consider a one-dimensional simple harmonic oscillator moving in a potential $V(x) = \frac{1}{2} m\omega^2 x^2$.

Given that the ground state wave function is $\psi(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} \exp\left(-\frac{1}{2}\alpha x^2\right)$ (where $\alpha = m\omega/\hbar$). Find the expectation value of (x^2).

(b) For a Hamiltonian $\hat{H} = (\hat{p}^2/2m) + V(\hat{x})$, prove that $\left[\hat{x}, \left[\hat{x}, \hat{H}\right]\right] = -\frac{\hbar^2}{m}$.

(c) Prove that $\exp\left[i(\hat{A}\hat{B} - \hat{B}\hat{A})\right]$ is a Hermitian operator, if $\hat{A}, \hat{B}$ are Hermitian operators. 4+3+3

9. (a) A stream of particles of mass m and energy E move towards the potential step $V(x) = 0$ for $x < 0$ and $V(x) = V_0$ for $x \geq 0$. If the energy of the particles $E < V_0$,

(i) show that there is a finite probability of finding the particles in the region $x > 0$.

(ii) sketch the solutions in the two regions.

(iii) determine the reflection coefficient and comment on the result.

(b) Write down Pauli's spin matrices σ_x, σ_y and σ_z. The eigenfunctions of the Pauli spin operator σ_z are α and β. Show that $\frac{\alpha+\beta}{\sqrt{2}}$ and $\frac{\alpha-\beta}{\sqrt{2}}$ are the eigenfunctions of σ_x. (3+1+2)+(2+2)

10. (a) Write down the Schrödinger equation for the hydrogen atom assuming the nucleus heavy. Obtain the radial part of the equation.

(b) In the ground state of hydrogen atom show that the probability P for the electron to lie within a sphere of radius R is

$$P = 1 - \exp\left(-\frac{2R}{a_0}\right)\left(1 + \frac{2R}{a_0} + 2R^2/a_0^{\,2}\right) \text{ where } \Psi(100) = (\pi a_0^{\,3})^{-1/2} \exp(-r/a_0).$$

(c) Write down the operators for L^2 and L_z in polar coordinates. Hence verify that $\Psi = A \sin\theta\, e^{i\phi}$, where A is a constant, is an eigenfunction of L^2 and L_z. Find the eigenvalues. 4+2+4

Section - II

(Atomic Physics)

Answer ***any two*** questions.

11. (a) In a Stern–Gerlach experiment, a beam of silver atoms moving with a velocity 'v' passes through an inhomogeneous magnetic field of gradient $\frac{\partial B}{\partial z}$ for a distance of 'l'. After emerging from the magnetic field, they travel a distance 'b' before reaching the screen. What will be the magnitude of the splitting?

(b) What is the g-factor for an atom with a single optical electron in $d_{\frac{3}{2}}$ level?

(c) Consider the L-S coupling scheme for helium atom. Show that (i) $1s^1 2s^1$ configuration leads to the terms ${}^1S_0$ and ${}^3S_1$ while (ii) $1s^1 2p^1$ configuration leads to ${}^1P_1$, ${}^3P_0$, ${}^3P_1$ and ${}^3P_2$.
4+2+(2+2)

12. (a) The spacing between the vibrational levels of CO molecule is 0.08 eV. Calculate the value of the force constant of the CO bond. Given that the masses of C and O atoms are 2.0×10^{-26} kg and 2.7×10^{-26} kg respectively. ($\hbar = 6.58 \times 10^{-16}$ eV sec)

(b) Do hydrogen molecules give rise to pure vibration-rotation spectra? Justify your answer.

(c) Pure rotational spectrum is almost always seen as absorption lines, and not as emission lines. Explain. 4+3+3

13. (a) Draw the energy level diagram for a four-level laser. Explain the requirement of each energy level. Why is a four-level laser perferred to a three-level laser?

(b) In a He-Ne laser transition from $3S$ to $2P$ level gives a laser emission of wavelength 632.8 nm. If the $2P$ level has energy equal to $15.2 \times 10^{-19} J$, assuming no loss, calculate the pumping energy required.

(c) Why do molecules show band spectra rather than line spectra? (2+3+1)+2+2

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2020

PHYSICS — HONOURS

Paper : CC-2

(Mechanics)

Full Marks : 50

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Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

1. Answer ***any five*** questions of the following : 2×5

(a) Show that mutually interacting forces on a system of particles have no effect on its total linear momentum.

(b) A solid sphere and a solid cylinder having same mass and same radii roll down an inclined plane without slipping. Show that the sphere will reach the bottom first.

(c) 'In streamline flow of a Newtonian fluid two streamlines never intersect'— Explain.

(d) Prove that the areal velocity of a particle moving under a central force field is constant.

(e) What is the rotational period of a binary star consisting of two equal masses, M and separated by distance L?

(f) Find the degrees of freedom of a system of two point masses joined by a massless rigid rod in a 3-dimensional space.

2. (a) A particle is moving in a plane in such a way that its polar co-ordinates are given by $r = 2t + 3$ and $\theta = 3t - t^2$. Obtain the radial and transverse components of instantaneous acceleration.

(b) A particle of mass 'm' at rest at $(a, 0, 0)$ subjected to a force $\vec{F} = -\frac{k}{x^3}\hat{x}$, where k is a positive constant. Find the time taken by the particle to reach the origin.

(c) Given $\vec{F} = -r\hat{r}$ is a conservative force field. Find the corresponding scalar potential. 4+4+2

3. A particle of mass m moves along a trajectory given by $x = x_0 \cos \omega_1 t$, $y = y_0 \sin \omega_2 t$, where x_0 and y_0 are constants.

(a) Find the x and y components of the force. What is the condition under which the force is a central one?

(b) Find the potential energy as a function of x and y.

(c) Determine the kinetic energy of the particle. Show that the total energy of the particle is conserved. (2+1)+3+(2+2)

Please Turn Over

4. (a) Show that the total angular momentum of a system of particles about any arbitrary point is the sum of angular momentum due to a single particle of total mass of the system situated at the centre of mass and the angular momentum of the particles about the centre of mass.

(b) Prove that total energy of a particle of mass '*m*' acted upon by a central force is given by,

$$E = \frac{L^2}{2m}\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right] + V(r)$$

where L is the angular momentum, $V(r)$ is the potential energy, $u = \frac{1}{r}$, r and θ being the polar co-ordinates. 5+5

5. (a) Show how a fictitious force arises in a non-inertial frame which is moving with a constant acceleration in a given direction with respect to a fixed frame.

(b) Let S' be a reference frame which is rotating with respect to a fixed frame S with an angular velocity $\vec{\omega}$. Prove that for an arbitrary vector $\vec{A}$,

$$\frac{d\vec{A}}{dt} = \frac{d'\vec{A}}{dt} + \vec{\omega} \times \vec{A}$$

where $\frac{d}{dt}$ and $\frac{d'}{dt}$ refer to time derivatives with respect to S and S' frames, respectively.

(c) Two reference frames, one is fixed and other one is rotating, have common origin. Obtain the equation of motion of a particle of mass '*m*' with respect to the rotating frame. Discuss about the different fictitious forces arise in the rotating frame. 2+4+4

6. (a) Show that the angular momentum vector $\vec{L}$ is not always along the same direction as the instantaneous axis of rotation.

(b) Determine the moment of inertia tensor for the configuration in which four point masses of 1, 2, 3 and 4 units are located at (1, 0, 0), (1, 1, 0), (1, 1, 1) and (1, 1, –1) units, respectively.

(c) A rigid body is rotating under the influence of an external torque $\vec{N}^{(e)}$. If the angular velocity is $\vec{\omega}$ and kinetic energy is T, show that

$$\frac{dT}{dt} = \vec{N}^{(e)} \cdot \vec{\omega}$$

when the axes of the body co-ordinates are taken as principal axes.

(d) Indicate the principal axes for a homogeneous sphere and a cylinder in neatly labelled sketches. 2+3+3+2

7. (a) Set up Euler's equation for an incompressible fluid and establish Bernoulli's equation of fluid motion stating the assumptions used. 6

(b) A pipe of varying diameter is used to lift water by 7m. The area of cross-section of the pipe at the base is 125 cm^2 and the pressure here is $2{\cdot}5\times10^5$ Pa. The area of cross-section of the pipe at the top is 25 cm^2. The rate of flow of water is 3×10^{-2} m^3/sec. Calculate the pressure of water at the top, neglecting energy losses. 4

Or,

A copper wire of diameter 1*mm*. and length 3*meters* has Young's modulus 12.5×10^{11} dynes per sq.cm., If a weight of 10*kg.* is attached to one end, what extension is produced? If the Poisson's ratio is 0.26, what lateral compression is produced? 4

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2020

PHYSICS — HONOURS

Fifth Paper

Full Marks : 100

The figures in the margin indicate full marks.

Candidates are required to give their answers in thier own words as far as practicable.

1. Answer ***any five*** of the following : 4×5

(a) Define generalized coordinates. A particle is moving on the surface of a sphere. Mention its generalized coordinates.

(b) Show that angular momentum is conserved for the Lagrangian $L = \dot{r}^2 + \vec{r}.\dot{\vec{r}} + r^2$.

(c) Calculate the Hamiltonian for a Lagrangian $L(x, \dot{x}) = \frac{1}{2}x\dot{x}^2 - V(x)$.

(d) Muons at their rest frame have lifetime $2{\cdot}3\times10^{-6}$s while those in cosmic shower have lifetime 16×10^{-6}s measured from earth. Find the speed of the muons in cosmic shower.

(e) If A_i and B_j are arbitrary covariant vectors and $c^{ij} A_i B_j$ is an invariant, prove that c^{ij} is a contravariant tensor of rank two.

(f) Does the density of an object change as its speed increases? If yes, by what factor?

(g) Find the eigenvalues and eigenfunctions of the angular momentum operator $\hat{L}_z = i\hbar\frac{\partial}{\partial\phi}$.

(h) Show that for a potential $V(-\vec{r}) = V(\vec{r})$, the eigen functions of the Hamiltonian must be of even or odd parity.

(i) A particle in a one-dimensional harmonic oscillator potential is described by a wave function $\psi(x, t)$. If the wave function changes to $\psi(\lambda x, t)$, then the expectation value of the kinetic energy T will change to what value?

(j) In a Stern-Gerlach experiment, a collimated beam of neutral atoms is split up into seven equally spaced lines. What is the total angular momentum of the atom?

(k) Explain why is pure vibrational spectra observed only in liquids.

(l) Find the magnetic dipole moment of the state $^2D_{3/2}$.

Group - A
Section - I
(Classical Mechanics II)

Answer ***any one*** question.

2. (a) State Kepler's laws of planetary motion.

(b) A particle of mass m moves under the action of a central force $f(r)\hat{r}$. Show that the equation determining the orbit of the particle is

$$\frac{l^2}{mr^2}\left(\frac{d^2(1/r)}{d\theta^2}+\frac{1}{r}\right)=-f(r)$$

where θ is the azimuthal angle and l is a constant of motion. Hence show that for an inverse square force, the trajectory is a conic section. 8+(8+4)

3. (a) State with reasons if the constraints in the following systems is Holonomic or not : (i) Two point masses connected by a massless rigid rod. (ii) The molecules of a gas within a container.

(b) The Lagrangian for a system is given by $L = \frac{1}{2}\dot{q}^2 - q\dot{q} + q^2$. Set up the Hamiltonian for the system and hence find the momentum p conjugate to q.

(c) The length of a plane simple pendulum changes with time such that $l = a + bt$ where a and b are constants. Find the Lagrangian equation of motion. Obtain the Hamiltonian and show that it is not invariant under time-translation i.e. $\frac{\partial H}{\partial t} \neq 0$. 6+6+8

4. (a) Set up Euler's equation of hydrodynamics for an incompressible fluid.

(b) An incompressible fluid of density ρ flows through a horizontal pipe of circular cross-section. The flow is constricted at one place where the radius is reduced from r_1 to r_2. If the difference of pressure before and after the constriction is Δp, show that the rate of flow of the liquid is

$$\sqrt{\frac{2\pi^2 r_2^4 r_1^4 \Delta p}{\rho\left(r_2^4 - r_1^4\right)}}$$

(c) What is a cyclic coordinate? Show that for each cyclic coordinate there exist a constant of motion. 6+8+6

Section - II
(Special Theory of Relativity)

Answer ***any one*** question.

5. (a) Define Lorentz invariant proper time interval $d\tau$.

(b) Find matrices (i) $[g_{ij}]$ and (ii) $[g^{ij}]$ corresponding to $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3$ consistent with the definition $ds^2 = g_{ij}dx^i dx^j$.

(c) Two lumps of clay each of rest mass m_0 move towards each other with equal speed $\frac{3}{5}c$ and stick together. What is the mass of the composite lump? 4+(4+4)+8

6. (a) Show that it is impossible for an isolated free electron to absorb or emit a photon.

(b) Show that the scalar product $A_\mu B^\mu$ of two four vectors A^μ, B^μ is invariant under Lorentz transformation.

(c) For a particle of rest mass m_0 and momentum p, show that the kinetic energy is given by

$$E = \sqrt{p^2c^2 + m_0^2c^4} - m_0c^2$$ 6+6+8

7. (a) Lagrangian of a one-dimensional, relativistic harmonic oscillator of rest mass m, is

$$L = mc^2\left(1 - \sqrt{1-\beta^2}\right) - \frac{1}{2}kx^2 .$$

Obtain the Lagrange equation of motion and show that it can be integrated to yield

$$E = mc^2 + \frac{1}{2}ka^2$$

where a is the maximum displacement from equilibrium of the oscillating particle.

(b) Calculate the radius of the orbit of an electron of energy E moving at right angles to an uniform magnetic field B.

(c) In a coordinate system with coordinates x^μ, the invariant line element is $ds^2 = \eta_{\alpha\beta}dx^\alpha dx^\beta$. If the coordinates are transformed $x^\mu \to \bar{x}^\mu$. Show that the line element is $ds^2 = g_{\bar{\mu}\bar{\nu}}d\bar{x}^\mu d\bar{x}^\nu$ and express $g_{\bar{\mu}\bar{\nu}}$ in terms of the partial derivatives $\partial x^\mu / \partial \bar{x}^\nu$. For two arbitrary 4– vectors U and V, show that,

$$U.V = U^\alpha V^\beta \eta_{\alpha\beta} = U^{\bar{\alpha}} V^{\bar{\beta}} g_{\bar{\alpha}\bar{\beta}}$$ (4+4)+4+(4+4)

Group - B

Section - I

(Quantum Mechanics II)

Answer ***any one*** question.

8. (a) The wave function of an excited state of a one-dimensional linear harmonic oscillator is of the form

$$\psi(x) = Axe^{-\alpha^2x^2/2}$$

(i) Calculate the expectation value of the momentum operator for this state.

(ii) Sketch the wave function and the corresponding probability density function.

(iii) Write down the energy eigenvalue for this state.

(b) A spin - 0 particle of mass 'm' is in a three dimensional isotropic well described by $V(r) = \frac{1}{2} m\omega^2 r^2$ where $r^2 = x^2 + y^2 + z^2$. How many states will have energy $\frac{7}{2}\hbar\omega$? (8+4+2)+6

9. (a) Let two spin $-\frac{1}{2}$ states of an electron are χ_α, χ_β. What will be the normalized singlet and triplet spin states formed by two such electrons?

(b) Evaluate the commutators (i) $[L_x^2 + L_y^2, L_z^2]$ and (ii) $[L_z, \sin 2\phi]$ (in usual notations).

(c) Find the energy eigenvalues and the normalized wave functions of a particle confined in a one-dimensional box extending over the region $-L/2 < x < L/2$. 6+(4+4)+6

10. (a) Find the expectation value of an operator $\hat{A}$ for a state $\psi = \frac{1}{\sqrt{2}}\phi_1 + \sqrt{\frac{2}{5}}\phi_2 + \frac{1}{\sqrt{10}}\phi_3$ where ϕ_1, ϕ_2 and ϕ_3 are three orthonormal eigen states of $\hat{A}$ such that $\hat{A}\phi_n = n^2\phi_n$.

(b) Consider a potential barrier

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \text{ and } x > a \\ V_0 & \text{for } 0 < x < a \end{cases}$$

where V_0 is positive. When a particle of mass m and energy $E(0 < E < V_0)$ approaches the barrier from the left, show that the transmission coefficient is

$$\left[1 + \frac{V_0^2 \sin h^2(\beta a)}{4E(V_0 - E)}\right]^{-1}$$

where $\beta = \sqrt{2m(V_0 - E)}/\hbar$. 8+12

Section - II

(Atomic Physics)

Answer ***any one*** question.

11. (a) Deduce an expression for the Lande'g factor of an atom.

(b) Write the spectral symbol of the term with $s = 1/2$, $j = 5/2$, and Lande g factor 6/7.

(c) The quantum numbers of two electrons in a two-valence electron atom are

$$n_1 = 6,\ l_1 = 3,\ s_1 = \frac{1}{2};\ n_2 = 5,\ l_2 = 1,\ s_2 = \frac{1}{2}.$$

(i) Assuming LS coupling, find the possible values of L and hence J.

(ii) Assuming JJ coupling, find possible values of J. 8+6+6

12. (a) The energy levels corresponding to the pure rotational spectrum of diatomic molecules is given by $E_j = BJ(J+1)$, where B is a constant. If λ_1 and λ_2 be the wavelengths of lines corresponding to transitions $J \rightarrow J+1$ and $J+1 \rightarrow J+2$ respectively, find B in terms of these wavelengths.

(b) Assuming that the hydrogen molecule behaves like a harmonic oscillator with a force constant $k = 573$ N/m, find the vibrational quantum number corresponding to its 4.50 eV vibrational energy level, given that the mass of the H atom $= 1.67\times10^{-27}$ kg and $h = 6.63\times10^{-34}$ Js.

(c) For normal Zeeman effect in hydrogen, explain how Lorentz triplet occurs. 6+8+6

13. (a) What is the role played by an optical resonator in a laser system?

(b) The relative population of two energy states at room temperature $T = 300$K is $1/e$. Determine the wavelength of radiation emitted due to transition between the states.

(c) Consider a 4-level system in which lasing transition occurs between the levels $2 \rightarrow 1$. Show that a necessary condition for population inversion is $\gamma_{10} > \gamma_{21}$ where γ stands for spontaneous decay rate and 0 denotes ground state.

(d) What is Raman effect? What is its practical application? 4+4+6+(4+2)

2018

PHYSICS – HONOURS

Fifth Paper

Full Marks – 100

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The figures in the margin indicate full marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer ***any ten*** of the following questions : 2×10

(a) Define holonomic and non-holonomic constraints with an example of each.

(b) If the Lagrangian is given by

$$L(x,\dot{x})=\frac{\dot{x}^2}{2x}-V(x)$$

What will be the corresponding Hamiltonian?

(c) An U-tube contains a zero-viscosity fluid up to a length L. If one end of the fluid column is depressed by a little distance, show that the fluid column oscillates with time period, $T=2\pi\sqrt{\frac{L}{2g}}$; g being the acceleration due to gravity.

(d) The momentum of a body becomes four times when its speed doubles. What was the initial speed of the body in units of c?

(e) Prove that the four dimensional volume element $dx\ dy\ dz\ dt$ is invariant under Lorentz transformation.

(f) If A_i and B_j are arbitrary covariant vectors and $C^{ij}A_iB_j$ is a scalar, prove that C^{ij} is a contravariant tensor of second rank.

(g) Commutator of two matrices A and B is defined by [A, B] = AB – BA and the anti- commutator by {A, B} = AB + BA. If {A, B} = 0, find [A, BC].

(h) Can we measure the kinetic and potential energies of a particle simultaneously with arbitrary precision?

(i) Show that the commutator $[x, [x, H]] = -\frac{\hbar^2}{m}$, where H is the Hamiltonian operator.

(j) Calculate the angle between the total $(\bar{J})$ and orbital angular $(\bar{L})$ momentum vectors for an electron in $^4D_{3/2}$ state.

(k) Which of the following substances can give rise to pure rotation-vibration spectra?

H_2, HF, O_2, CO

(l) What is the importance of presence of a metastable state in lasing action?

Group – A

Section – I

Answer ***any two*** questions

2. (a) Show that the effective potential of a particle of mass m in a central force field is given by

$$U_{eff}(r) = U(r) + \frac{L^2}{2mr^2}$$

where L is the angular momentum. 3

(b) A particle of mass m moving in a central force field describes a spiral orbit $r = k\theta^2$ where k is a positive constant.

(i) Find the force law. Given that the differential equation of the orbit is

$$\frac{L^2}{mr^2}\left(\frac{d^2\left(\frac{1}{r}\right)}{d\theta^2} + \frac{1}{r}\right) = -f(r).$$

(ii) Compute the effective one-dimensional potential energy.

(iii) Find the total energy of the system, for which this motion is allowed.

2+3+2

3. (a) What do you mean by generalized forces? Find its expression in terms of generalized coordinates. 1+2

(b) A particle of mass m is constrained to move on the surface of a smooth sphere of radius R. There are no external forces of any kind acting on the particle.

(i) Choose a set of generalized coordinates and write the Lagrangian of the system.

(ii) Derive the Hamiltonian of the system. Is it conserved?

(iii) Using the Hamiltonian equations of motion, prove that the motion of the particle is along a great circle of the sphere. [A great circle on a sphere is a circle on the sphere's surface whose center is the same as the center of the sphere].

2+(2+1)+2

4. (a) Consider the Lagrangian

$$L = \frac{1}{2}m\left(\dot{x}^2 - \omega^2 x^2\right)e^{\gamma t}$$

for the motion of a particle of mass m in one-dimension (x). The constants m, γ and ω are real and positive. Construct the Hamiltonian. Is the Hamiltonian a constant of motion? 3+1

(b) Show that $q_1 q_2$ is a constant of motion for the Hamiltonian $H = q_1 p_1 - q_2 p_2 - aq_1^2 + bq_2^2$. 2

(c) Establish Bernoulli's equation of fluid motion stating the assumptions used. 4

Section – II

Answer ***any two*** questions

5. (a) From the basic definition of space-time interval, explain with suitable diagrams (i) a space-like interval, and (ii) a time-like interval for a two-dimensional space-time geometry. 2+2

(b) In a certain inertial frame light pulses are emitted by two sources 5 km apart. Time interval between two pulses is $5\,\mu s$. An observer moving at a speed V along the line joining these sources notes that the pulses are simultaneous. Find the speed V of the observer. 3

(c) Two rockets of rest length L_0 are approaching each other from opposite directions at same speed $\frac{c}{2}$. How long does one of them appear to the other? 3

6. (a) Define proper time interval and show that it is a Lorentz invariant quantity. Hence construct velocity four-vector and show that it is a time like vector. 1+1+1+1

(b) A neutral pion of rest mass m and relativistic momentum $P = \frac{3}{4}mc$ decays into two photons. One of the photons is emitted in the same direction as the original pion, and the other in the opposite direction. Find the relativistic energy of each photon. 4

(c) Discuss about inconsistency, if any, in Newton's law of gravitation in the light of postulates of special theory of relativity. 2

7. (a) Defining $A_j = g_{jk}A^k$ and $A^k = g^{jk}A_j$ where symbols bear usual meaning, write down the mathematical relationship between g^{jk} and g_{jk}. What are the signed values of g^{ij} and g_{ij} in case of a 3 dimensional flat space time (2 space and 1 time dimension)? 2+2

(b) What is the space-time interval between any two events on the locus of a photon in space-time geometry; consistent with the definition $ds^2 = g_{ij}dx^i dx^j$? How does this interval vary from one inertial frame to another? Explain your answer with justification. 2

(c) A particle of rest mass m_0 moving with speed V collides and sticks with a stationary particle of rest mass M_0. Show that the speed of the composite particle is given by

$$\gamma m_0 V/(M_0 + \gamma m_0), \text{ where } \gamma = \left(1 - \frac{V^2}{C^2}\right)^{-\frac{1}{2}}.$$ 4

Group – B

Section – I

Answer ***any two*** questions

8. (a) Consider the potential $V(x) = \begin{cases} 0, & 0 < x < a \\ \infty, & \text{elsewhere} \end{cases}$.

(i) Estimate the energies of the ground state and the first excited state for an electron enclosed in a box of size $a = 10^{-10}$ m.

(ii) Calculate the same energies for a 1g metallic sphere which is moving in a box of size a = 10 cm.

(iii) From results of (i) and (ii) discuss why quantum mechanical effects are not important in second case. 4+2+1

(b) For the above potential, the wave function of a particle in the position space is given by $\phi(x) = \sqrt{\frac{2}{a}} \sin\frac{2\pi x}{a}$. Determine its momentum space wave function. 3

9. (a) Show that for stationary states in quantum mechanics variance of H (Hamiltonian) is zero. 2

(b) A potential barrier of height V_0 extends from $x = -a$ to $x = +a$. Prove that for a particle of energy $E < V_0$, the transmission coefficient through the barrier is given by

$$T^{-1} = 1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2\left(\frac{2a}{\hbar}\sqrt{2m(V_0 - E)}\right)$$ 5

(c) The ground state wave function of a particle of unit mass moving in a one-dimensional potential $V(x)$ is $e^{-\frac{x^2}{2}} \cosh\left(\sqrt{2}x\right)$. Find the potential $V(x)$, in suitable units in which $\hbar = 1$. 3

10. (a) A harmonic oscillator has a normalized wave function $\psi(x) = \frac{1}{\sqrt{3}}\psi_2(x) + c\psi_7(x)$ where $\psi_n(x)$ are the energy eigen-states. What is the magnitude of the constant c? Hence find the expectation value of the energy in this state. 2+2

(b) Write down Pauli's spin matrices σ_x, σ_y and σ_z. The eigen-vectors of the operator σ_z are $|\alpha\rangle$ and $|\beta\rangle$. Show that $\frac{|\alpha\rangle+|\beta\rangle}{\sqrt{2}}$ and $\frac{|\alpha\rangle-|\beta\rangle}{\sqrt{2}}$ are the normalised eigen-vectors of σ_x. 2+2

(c) Evaluate the following commutator :

$$\left[\vec{L}.\vec{S}, J^2\right].$$ 2

Section – II

Answer ***any two*** questions

11. (a) Calculate the Lande' g factors for $^3S_1$ and $^3P_1$ levels. Hence estimate the energy splitting of the two levels if a magnetic field of 1 T is applied. How many spectral lines will arise from the anomalous Zeeman splitting due to transition between these levels? Draw a neat diagram showing these transitions. (One Bohr Magneton = 9.27×10^{-24} J/T) 2+2+3

(b) The $J=0\rightarrow J=1$ rotational absorption line occurs at 1.153×10^{11} Hz in $C^{12}O^{16}$ and at 1.102×10^{11} Hz in C^xO^{16}. Find the mass number of the unknown carbon isotope (C^x). 3

12. (a) Into how many fine structure lines each line of the Balmer series of Hydrogen will split due to spin-orbit coupling? Justify your answer. 4

(b) Consider a two-electron system with $l_1=2$ and $l_2=1$. What are the possible total angular momentum J states, assuming LS coupling? Write the spectral term for each state. 4

(c) In a Stern-Gerlach experiment, always a beam of neutral atoms is used, and not ions. Explain the reason. 2

13. (a) Show that in a 3-level system it is possible to produce the required population inversion using a beam of suitable intensity. What is inversion threshold pumping rate? 4+2

(b) In a He-Ne laser, transition from 3S to 2P level gives a laser emission of wavelength 632.8 nm. If the 2P level has energy equal to 15.2×10^{-19} J, assuming no loss, calculate the pumping energy required. 2

(c) Why do molecules show band spectra rather than line spectra? 2