

2025

## PHYSICS — HONOURS — PRACTICAL

Paper : DSCC-7P

(Mathematical Physics - II)

Full Marks : 25

*The figures in the margin indicate full marks.*

Day - I

Program = 15, CNB = 5, Viva = 5

Answer *any one* question.

1. (a) A simple pendulum consists of a bob of mass  $m$  attached to a string of length  $L$ . Ignoring air resistance and assuming the motion is confined to a plane, the equation of motion for the pendulum is given by :

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$$

Write a Python code using `scipy` to numerically solve this equation for the following conditions : length of pendulum  $L = 1\text{m}$ , acceleration due to gravity  $g = 9.81\text{ m/s}^2$ , initial angle  $\theta(0) = \pi/4$ , initial angular velocity  $\frac{d\theta}{dt}(0) = 0$ . Simulate the motion for  $t \in [0, 10]$  seconds. Plot the angular displacement vs time with appropriate labels. [Code : 6, Plot : 2]

- (b) You are given a set of experimental data points representing the temperature recorded at different times as follows :

Time (in minutes)	0	5	10	15	20	25
Temperature (in °C)	30	35	50	65	80	85

Using suitable interpolation function from `scipy` library, estimate the temperature at 12 minutes, 18 minutes and 22 minutes. Plot the original data points and the interpolated values on the same graph. Label the axes and include a legend. [Code: 3, Output: 2, Plot: 2]

(2)

D(4th Sm.)-Physics-II/Pr./DSCC-7P/CCF(Day-I)

2. (a) Compute the value of  $\pi$  from the formula :

$$\frac{\pi}{4} = \int_0^1 \frac{dx}{1+x^2}$$

Use Quad function and Simpson function from scipy library. Print the absolute difference of the value of  $\pi$  using the Quad function and the Simpson function. Plot the integrand in the region  $x \in [0, 1]$ . **[Code : 5, Output : 1, Plot : 1]**

- (b) Numerically verify the recursion relation of the given Legendre's polynomial.

$$(1-x^2)P_n'(x) = (n+1)xP_n(x) - (n+1)P_{n+1}(x)$$

$x \in [-1, 1]$ . Take  $n = 2, 4$ . Plot the right hand side and left hand side of the given relation in the same graph. **[Code : 6, Plot : 2]**

3. (a) The Dirac delta function, denoted by  $\delta(x)$ , has the following sifting property :

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a)$$

Since  $\delta(x)$  cannot be represented directly in numerical computation, we can use a normalized Gaussian function to approximate it as :

$$\delta(x-a) = \frac{1}{\epsilon\sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\epsilon^2}}$$

Write a Python program using appropriate function from scipy to numerically verify the above sifting property using this approximation. Take  $f(x) = \sin(x)$ ,  $a = 1.0$  and  $\epsilon = 0.01$ . Print the integral value as well as the exact value  $f(a)$ . **[Code : 4, Verification : 3]**

- (b) Consider the following dataset :

x	0	1	2	3	4	5	6	7	8	9	10
y	2.0	2.7	3.8	5.4	7.3	10.0	13.5	18.2	24.4	32.6	43.2

Fit an exponential model of the form

$$f(x) = a \cdot e^{bx}$$

to the given data using the curve\_fit function from the scipy.optimize module. Determine the best-fit parameters  $a$  and  $b$ . Then, plot the fitted curve along with the original data points using matplotlib. Ensure the plot includes : appropriately labelled axes, a title, a legend indicating the data and the fitted curve. **[Code : 5, Output : 1, Plot : 2]**

(3262)

(3)

D(4th Sm.)-Physics-H/Pr./DSCC-7P/CCF(Day-I)

4. (a) A periodic square wave function is defined over the interval  $[-\pi, \pi]$  with a period of  $2\pi$ , as :

$$f(t) = \begin{cases} 1 & 0 < t < \pi \\ -1 & -\pi < t < 0 \end{cases}$$

The Fourier series of the function is given by :

$$f(t) \sim \sum_{n=1}^{\infty} b_n \sin(nt) \quad \text{where} \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(nt) dt$$

Use suitable functions from scipy to compute the coefficients  $b_n$  and then reconstruct the function using first 20 terms. Plot the original function as well as the Fourier series approximation in a same graph. Use appropriate labels and legend. **[Code : 6, Plot : 2]**

- (b) Solve :  $y' + 2y = 2 - e^{-4t}$ ,  $y(0) = 1$ , by using odeint function from scipy package. Compare with exact results :  $y(t) = 1 + 0.5e^{-4t} - 0.5e^{-2t}$ . Plot the numerical solution and the exact solution for  $0 < t < 5$  on the same graph by matplotlib. Label the  $x$  and  $y$  axes as 'Time' and 'Displacement' respectively. **[Code : 5, Plot : 2]**

5. Write a program to solve the 1D Diffusion equation

$$\frac{\partial U(x,t)}{\partial t} = D \frac{\partial^2 U(x,t)}{\partial x^2}$$

$$D = 1.0$$

$$U(x,0) = \sin x, \quad 0 \leq x \leq \pi \quad \text{Initial Condition}$$

$$U(0,t) = U(\pi,t) = 0, \quad \forall t \quad \text{Boundary Condition}$$

Space domain :  $x \in [0, \pi]$ , Time Domain :  $t \in [0, 2.0]$

Plot  $U(x, t)$  with respect to  $x$  for three different time instants.

**[Code : 12, Plot : 3]**

2025

## PHYSICS — HONOURS — PRACTICAL

Paper : DSCC-7P

(Mathematical Physics - II)

Full Marks : 25

*The figures in the margin indicate full marks.*

Day - II

Program = 15, CNB = 5, Viva = 5

Answer *any one* question.

1. (a) You are given the following data for a function :

$x$	0	1	2	3	4	5
$f(x)$	0	0.8415	0.9093	0.1411	-0.7568	-0.9589

Using cubic interpolation from scipy module, estimate the function values at  $x = 2.5$  and  $4.5$ . Plot the original data points and the interpolated values in a same graph with appropriate labels and legend.

[Code : 3, Output : 2, Plot : 2]

- (b) The Dirac delta function can be approximated as :

$$\delta(x-a) = \frac{1}{\pi} \frac{\epsilon}{(x-a)^2 + \epsilon^2}$$

This function has area 1 under the curve and becomes sharply peaked as  $\epsilon \rightarrow 0$ , mimicking the behaviour of the Dirac delta. Define a function  $f(x) = e^{-x^2}$  and take  $a = 0.5$  and  $\epsilon = 0.01$ . Write a Python script to numerically verify :

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a)$$

[Code : 5, Verification : 3]

(2)

2. (a) (i) Plot the first four Hermite polynomials using appropriate package from scipy.  
 (ii) Verify the orthogonality property of the Hermite polynomial

$$\int_{-\infty}^{\infty} e^{-x^2} H_n(x) H_m(x) dx = 2^n \sqrt{\pi} n! \delta_{mn}$$

where  $H_m$  is the  $m^{\text{th}}$  order Hermite polynomial. Choose two different pairs of integers ( $m, n$ ), one with  $n = m$  and another with  $n \neq m$ . **[Plot : 2, Code : 6]**

- (b) Evaluate the Integral for  $n = -1/2, 1/2, 1, 3/2, 5/2$ .

$$I(n) = \int_0^{\infty} e^{-x} x^{n-1} dx$$

Use any appropriate scipy package. Print the outputs.

**[Code : 5, Output : 2]**

3. (a) Solve using odeint function from scipy package.

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = \frac{e^x}{x}; \quad y(1) = 0, \quad y'(1) = 1$$

Exact analytical solution :  $y(x) = e^{x-1}(-1 + e + x - ex + ex \ln x)$ .

Plot the numerical solution and exact solution in the same graph.

**[Code : 6, Plot : 2]**

- (b) You are given a set of experimental data that represents the decay of a radioactive substance over time as :

$t$ (in second)	0	1	2	3	4	5	6	7	8	9	10
Count ( $N$ )	1000	818	670	548	447	367	301	246	200	162	131

The decay approximately follows the exponential law :

$$N(t) = N_0 e^{-\lambda t}$$

$N(t)$  is the quantity of substance at time  $t$ ,  $N_0$  is the initial quantity at  $t = 0$  and  $\lambda$  is the decay constant. Use suitable function from scipy to fit the model to the data. Print the best-fit values of  $N_0$  and  $\lambda$  (you should get the value of  $N_0$  close to the initial data as given). Plot both the original data and the fitted curve on the same graph using appropriate labels and legend.

**[Code : 4, Output : 1, Plot : 2]**

4. (a) A force  $F(x) = 5x^2$  N acts on a particle along the  $x$ -axis from  $x = 0$  to  $x = 3$  meters. Write a Python program using integrating function from `scipy` to calculate the work done by this force over the given displacement. Print the result. **[Code : 4, Output : 1]**
- (b) The Fourier expansion of a given function  $F(x)$  periodic over interval  $[-l, l]$  can be written as

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right) \right)$$

where  $a_n = \frac{1}{l} \int_{-l}^l f(x) \cos\left(\frac{n\pi x}{l}\right) dx, n = 0, 1, 2, 3, \dots, \infty$

$$b_n = \frac{1}{l} \int_{-l}^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx, n = 1, 2, 3, \dots, \infty$$

Analyse the Fourier series of a Sawtooth wave taking  $l = 1$ . Evaluate the Fourier coefficients and print first 10 coefficients. Plot the constructed series with the given periodic function.

**[Code : 7, Plot : 3]**

5. Write a program to solve Laplace's equation on square grid

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

Dirichlet Boundary conditions are :

Left B.C. :  $U(0, y) = \frac{y}{1+y^2}$  for  $0 < y < 1$

Right B.C. :  $U(1, y) = \frac{y}{4+y^2}$  for  $0 < y < 1$

Bottom B.C. :  $U(x, 0) = 0.0$  for  $0 < x < 1$

Top B.C. :  $U(x, 1) = \frac{1}{(1+x)^2 + 1}$  for  $0 < x < 1$

The exact analytical solution is  $\frac{y}{(1+x)^2 + y^2}$ . Plot the numerical solution and exact analytical solution

with respect to  $x$  keeping  $y$  fixed at 0.4.

**[Code : 12, Plot : 3]**

2025

## PHYSICS — HONOURS — PRACTICAL

Paper : DSCC-7P

(Mathematical Physics - II)

Full Marks : 25

*The figures in the margin indicate full marks.*

Day - III

Program = 15, CNB = 5, Viva = 5

Answer *any one* question.

1. (a) In a physics lab experiment, the intensity of light  $I$  passing through a polarizer and analyzer is measured as a function of the angle  $\theta$  between their axes. According to Malus' Law, the intensity of light transmitted through the analyzer is given by :

$$I(\theta) = I_0 \cos^2(\theta - \theta_0)$$

where  $I(\theta)$  is the measured intensity at angle  $\theta$  (in degrees),  $I_0$  is the maximum intensity and  $\theta_0$  is the angle at which maximum intensity occurs. You are provided with the following data from the experiment :

$\theta$ (degrees)	0	15	30	45	60	75	90
$I$ (intensity)	0.99	0.94	0.76	0.52	0.29	0.12	0.01

Use suitable function from scipy to fit the Malus' Law to the given data. Print the best-fit parameters  $I_0$  and  $\theta_0$ . Plot the original data and the fitted curve on the same graph using appropriate labels and legend.

**[Code : 4, Output : 1, Plot : 2]**

- (b) Using special function from scipy, write a Python script to generate and plot the first five Legendre polynomials  $P_0(x)$  to  $P_4(x)$  over the interval  $x \in [-1, 1]$ . Use appropriate labels and legend. Verify the orthogonality relation :

$$\int_{-1}^1 P_n(x) P_m(x) dx = \frac{2}{2n+1} \delta_{mn}$$

numerically using suitable function from scipy library. Take the cases (i)  $m = n = 2$  and (ii)  $m = 3, n = 4$ .

**[Code : 4, Plot : 2, Verification : 2]**

2. (a) Verify the relation :

$$\int_{-\infty}^{\infty} \delta(x-a)f(x)dx = f(a)$$

Limiting representation of Dirac-Delta function :  $\delta_n(x-a) = \lim_{n \rightarrow \infty} \sqrt{\frac{n}{\pi}} e^{-n(x-a)^2}$ .

Take,  $f(x) = x$  and  $a = 2$ . Plot  $\delta_n(x-a)$  vs  $x$  for three values of  $n$ . Set appropriate axes labeling and put legend in the graph. [Code : 5, Plot : 2]

- (b) In an LCR circuit with no external voltage source (free oscillation), the charge  $q(t)$  on the capacitor satisfies the second-order differential equation :

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = 0$$

Given : Inductance  $L = 1$  H, resistance  $R = 0.5 \Omega$ , capacitance  $C = 0.01$  F with initial charge

$q(0) = 1$  C and initial current  $I(0) = \frac{dq}{dt}(0) = 1$  A. Write Python code to convert the second-order

ODE into a system of first-order ODEs and solve it numerically using suitable function from scipy. Plot the charge  $q(t)$  and current  $I(t)$  vs.  $t$ . [Code : 6, Plot : 2]

3. (a) The following table provides the distance to the visible horizon (in nautical miles) as a function of the observer's height above the Earth's surface (in feet) :

Height (x) [ft]	100	150	200	250	300	350	400
Distance (y) [nautical miles]	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Use suitable function from scipy to estimate the distance to the horizon when the height is  $x = 180$  feet using Lagrange interpolation. Plot the given data points and the interpolated polynomial using matplotlib. Highlight the interpolated point. Label the axes appropriately and include a title **Distance-Height Graph**. [Code : 4, Output : 1, Plot : 2]

- (b) Write a Python program using function from scipy to numerically verify :

$$\int_{-\infty}^{\infty} e^{-ax^2+bx} dx = \sqrt{\frac{\pi}{a}} \exp\left(\frac{b^2}{4a}\right)$$

Show your results for two different combinations of  $a$  and  $b$  :  $a = 1, b = 0.5$  and  $a = 1, b = 2$ . Plot the integrand as function of  $x$  for any one combination of  $a, b$ .

[Code : 4, Verification : 2, Plot : 2]

4. (a) Evaluate the Integral and print the output :

$$\int_0^{\infty} \sin(x^2)$$

Plot the integrand for  $0 \leq x \leq 10$ .

[Code : 4, Plot : 2]

- (b) The Fourier expansion of a given function  $F(x)$  periodic over interval  $[-l, l]$  can be written as

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right) \right)$$

where  $a_n = \frac{1}{l} \int_{-l}^l f(x) \cos\left(\frac{n\pi x}{l}\right) dx, \quad n = 0, 1, 2, 3, \dots, \infty$

$$b_n = \frac{1}{l} \int_{-l}^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx, \quad n = 1, 2, 3, \dots, \infty.$$

Write a program to compute the first 5 Fourier's coefficients ( $a_n, b_n$ ) of a square wave. Plot the constructed series and the square wave signal in the same graph using matplotlib. Set appropriate axis labeling and legend at suitable location in the graph. [Code : 5, Output : 2, Plot : 2]

5. Write a Python code to numerically solve the following equation using the finite difference method :

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0$$

with the initial and boundary conditions :

$$u(x, 0) = \sin(\pi x), \quad \frac{\partial u(x, 0)}{\partial t} = 0$$

$$u(0, t) = 0, \quad u(1, t) = 0$$

Take the spatial step-size  $\Delta x = 0.01$ , time step  $\Delta t = 0.005$  and final time  $t_{\max} = 1$ . Plot the wave profile  $u(x, t)$  at different time steps :  $t = 0, 0.25, 0.5, 0.75, 1.0$ . [Code : 12, Plot : 3]

2023

PHYSICS — HONOURS — PRACTICAL

Paper : CC-8P

(Syllabus : 2019-2020 and 2018-2019)

Full Marks : 30

DAY - 1

[Experiment : 20, Laboratory Notebook : 5, Viva voce : 5]

1. (a) The heat equation in one dimension is given by  $K \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial u(x,t)}{\partial t}$ ;  $K = \text{Constant}$ ,

where  $u(x, t)$  is the temperature function. Consider a wire of length  $0 \leq x \leq \pi$ . The boundary conditions at both ends are :

$$u(0, t) = 0 = u(\pi, t).$$

The initial temperature distribution is :

$$u(x, 0) = \sin x.$$

Write a Python program to solve the heat equation for the above system and plot the temperature distribution  $u(x, t = 0.5\text{s})$ . Take  $K = 0.05$  unit.

[Code : 10 + Plot : 2, Total : 12]

- (b) Bessel function of first kind of integer order with real value satisfy the relation :

$$J_{-n}(x) = (-1)^n J_n(x).$$

Using SciPy special module, write a Python program to plot  $J_n(x)$  and  $J_{-n}(x)$  in the same window for  $n = 1$  and  $n = 2$  in the range  $0 \leq x \leq 10$ . Comment how the above relation is satisfied from the plot.

[Code : 5 + Plot : 2 + Comment : 1, Total : 8]

2. (a) Compute the improper integral using quad function from SciPy package

$$I = \int_0^1 \frac{1}{\sqrt{1-x^2}} dx$$

Plot the integrand in the range  $x \in [0, 0.98]$ . Label the axes and write the integrand as a legend inside the graph.

[Program : 4 + Output : 1 + Graph : 1+½+½+1, Total : 8]

- (b) Write a program to solve Laplace's equation on square grid

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

Dirichlet Boundary conditions are :

Left B. C.  $U(0, y) = \frac{y}{1+y^2}$ , for  $0 < y < 1$

Right B. C.  $U(1, y) = \frac{y}{4+y^2}$ , for  $0 < y < 1$

Bottom B. C.  $U(x, 0) = 0$ , for  $0 < x < 1$

Top B. C.  $U(x, 1) = \frac{1}{(1+x)^2 + 1}$ , for  $0 < x < 1$

The exact analytical solution is  $\frac{y}{(1+x)^2 + y^2}$ . Plot the numerical solution ( $U_N$ ) and exact analytical solution ( $U_A$ ) with respect to  $x$  for a fixed  $y$  and with respect to  $y$  for a fixed  $x$ .

[Program : 8 + Graph : 2+2, Total : 12]

3. (a) (i) Plot Legendre's polynomial of orders  $n = 1, 2, 3$  (ii) Verify the orthogonality relation :

$$\int_{-1}^1 P_n(x)P_m(x)dx = \frac{2}{2n+1}\delta_{mn}, \text{ for } m, n = 1, 2,$$

where  $P_n$  is the  $n$ th order Legendre's polynomial. [Plot : 4 + Program : 6, Total : 10]

- (b) Consider the following differential equation which describes damped harmonic motion :

$$\frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + kx = 0, \quad x(0) = 0, \quad \frac{dx}{dt}(0) = 0$$

Consider  $\gamma = 0.5$  unit and  $k = 1$  unit. Write a Python program to solve the differential equation by using SciPy module. Plot  $x$  as a function of  $t$  for the range  $0 \leq t \leq 10$ .

[Code : 8 + Plot : 2, Total : 10]

4. (a) Solve using odeint package from SciPy :

$$\frac{d^2 y}{dx^2} + y = 4x + 10 \sin x, \quad y(\pi) = 0, y'(\pi) = 2.$$

Space domain :  $x \in [\pi, 4\pi]$ .

The exact solution is  $y = 9\pi \cos x + 7 \sin x + 4x - 5x \cos x$ . Plot the numerical solution and exact solution in the same graph.

[Program : 8 + Graph : 2, Total : 10]

- (b) The fourier expansion of a given function  $F(x)$  periodic over interval  $[-l, l]$  can be written as

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right) \right),$$

where  $a_n = \frac{1}{l} \int_{-l}^l f(x) \cos\left(\frac{n\pi x}{l}\right) dx, n = 0, 1, 2, 3, \dots, \infty$

$$b_n = \frac{1}{l} \int_{-l}^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx, n = 1, 2, 3, \dots, \infty.$$

Analyse the Fourier series of a Sawtooth wave. Evaluate the Fourier coefficients and print first 10 coefficients. Plot the constructed series with the periodic function taken.

[Program : 7 + Graph : 3, Total : 10]

5. (a) The Fourier expansion of a given function  $F(x)$  periodic over interval  $[-l, l]$  can be written as

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right) \right),$$

where  $a_n = \frac{1}{l} \int_{-l}^l F(x) \cos\left(\frac{n\pi x}{l}\right) dx, n = 0, 1, 2, 3, \dots, \infty$

$$b_n = \frac{1}{l} \int_{-l}^l F(x) \sin\left(\frac{n\pi x}{l}\right) dx, n = 1, 2, 3, \dots, \infty.$$

Write a program to compute the first ten fourier's coefficients ( $a_n, b_n$ ) of a square wave. Plot the constructed series and the square wave signal in the same graph using matplotlib. Set appropriate axis labelling and legend at suitable location in the graph.

[Program : 6 + Graph : (1+1)+(1+1), Total : 10]

- (b) Using SciPy module, write a Python program to solve the differential equation describing the motion of a particle under gravity subjected to a resistive drag force :

$$\frac{dv}{dt} = g - kv, \text{ where } v = \frac{dx}{dt}, \text{ 'x' being the displacement of the particle.}$$

Consider  $k = 0.01$  unit and  $v(t = 0) = 0$ . Plot  $v(t)$  vs.  $t$  for  $0 \leq t \leq 10$ s.

[Code : 8 + Plot : 2, Total : 10]

6. (a) Write a Python program using SciPy to show the convolution of the following two Gaussian functions is also a Gaussian :

$$f(t) = \frac{1}{\sqrt{\pi}} e^{-(t-2)^2}; \quad g(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$$

Plot the functions  $f(t)$ ,  $g(t)$  and the convoluted function on the same graph.

$$\text{Note : } (f * g)(t) := \int_{-\infty}^{\infty} f(\tau)g(t-\tau) d\tau$$

[Code : 7 + Plot : 3, Total : 10]

- (b) Write a program to solve 1d wave equation :

$$\begin{aligned} \frac{\partial^2 U}{\partial t^2} &= \frac{\partial^2 U}{\partial x^2} \\ U(0,t) = U(1,t) &= 0, \quad t > 0 \\ U(x,0) &= \sin(\pi x), \quad 0 \leq x \leq 1. \\ \left( \frac{\partial U}{\partial t} \right)_{x,t=0} &= 0, \quad 0 \leq x \leq 1 \end{aligned}$$

Plot the numerical  $U_N(x, t)$  solution in a graph with respect to  $x$  for different time instants ( $0 < t \leq 1.5$ ). Label the axes and put appropriate legends in the graph.

[Program : 7 + Graph : 1+(1+1), Total : 10]

7. (a) Verify the relation :

$$\int_{a-x_1}^{a+x_2} \delta(x-a) f(x) dx = f(a)$$

Limiting representation of Dirac-Delta function :  $\delta_n(x-a) = \lim_{n \rightarrow \infty} \sqrt{\frac{n}{\pi}} e^{-n(x-a)^2}$ .

Take,  $f(x) = x^2$  and  $a = 4$ . Plot  $\delta_n(x-a)$  vs.  $x$  for three values of  $n$ . Set appropriate axes labelling and put legend in the graph.

[Program : 6 + Graph : 2+2, Total : 10]

(b) Numerically verify the recursion relation of the given Legendre's polynomial :

$$(1-x^2)P_n'(x) = (n+1)xP_n(x) - (n+1)P_{n+1}(x)$$

$x \in [-1, 1]$ . Take  $n = 2, 4, 6$ . Plot the right hand side and left hand side of the given relation in the same graph.

[Program : 6 + Graph : 4, Total : 10]

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2023

PHYSICS — HONOURS — PRACTICAL

Paper : CC-8P

(Syllabus : 2019-2020 and 2018-2019)

Full Marks : 30

DAY - 2

[Experiment : 20, Laboratory Notebook : 5, Viva voce : 5]

1. (a) Bessel function of first kind satisfy the recurrence relation :

$$J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x).$$

Use the above relation to write a Python program, by using only  $J_0$  and  $J_1$  from SciPy, to calculate and plot  $J_2(x)$  and  $J_3(x)$  for  $0 \leq x \leq 20$ .

[Code : 7 + Plot : 3, Total : 10]

- (b) Write a program to solve the 1D Diffusion equation :

$$\frac{\partial U(x,t)}{\partial t} = D \frac{\partial^2 U(x,t)}{\partial x^2}$$

$$D = 1.0$$

$$U(x,0) = 0, \quad 0 \leq x \leq 10 \quad \text{Initial Condition}$$

$$U(0,t) = 0; \quad U(10,t) = 100, \quad \forall t \quad \text{Boundary Condition}$$

Space Domain :  $x \in [0, 10]$ , Time Domain :  $t \in [0, 5.0]$

Plot  $U(x, t)$  with respect to  $x$  for different time instants.

[Program : 8 + Graph : 2, Total : 10]

2. (a) Differential equation for a swinging pendulum is  $\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$ .

Write a suitable Python program using odeint to solve the above equation. Plot  $\theta$  as a function of  $t$  for  $0 \leq t \leq 5$ . The initial conditions are given as :  $\theta(t=0) = \frac{\pi}{6}$ ,  $\theta'(t=0) = 0$ . Take  $L = 10$  cm,  $g = 980$  cm/s<sup>2</sup>.

[Code : 10 + Plot : 2, Total : 12]

- (b) Write a Python program to numerically evaluate the following improper integral :

$$I(N) = \int_{-N}^N x^2 e^{-x^2} dx.$$

Increase 'N' gradually, so that  $I(N)$  converges to  $\sqrt{\pi}/2$ . Plot  $I(N)$  vs.  $N$  to show the convergence.

[Code : 6 + Plot : 2, Total : 8]

3. (a) Write a program to solve Laplace's equation on square grid :

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0.$$

Dirichlet Boundary conditions are :

Left B. C.  $U(0, y) = 0.0$  for  $0 < y < 1$

Right B. C.  $U(1, y) = y$  for  $0 < y < 1$

Bottom B. C.  $U(x, 0) = 0.0$  for  $0 < x < 1$

Top B. C.  $U(x, 1) = x$  for  $0 < x < 1$

The exact analytical solution is  $U_A(x, y) = xy$ . Plot the numerical solution ( $U_N$ ) and exact analytical solution ( $U_A$ ) with respect to  $x$  for a fixed  $y$  and with respect to  $y$  for a fixed  $x$ .

[Program : 8 + Graph : 2+2, Total : 12]

- (b) Compute the improper integral using quad function from SciPy package :

$$I = \int_3^5 \frac{x^2}{\sqrt{(x-3)(5-x)}} dx.$$

Plot the integrand in the range  $x \in [3.001, 4.999]$ . Label the axes and write the integrand as a legend inside the graph.

[Program : 4 + Output : 1 + Graph : 1+½+½+1, Total : 8]

4. (a) Dirac-Delta function can be expressed as the limit of a Gaussian distribution :

$$\delta(x - x_0) = \lim_{\sigma \rightarrow 0} G_{\sigma}(x - x_0),$$

$$\text{where } G_{\sigma}(x - x_0) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x - x_0)^2}{2\sigma^2}\right\}.$$

Take  $x_0 = 1$ . Plot  $G_{\sigma}(x - x_0)$  for  $\sigma = 0.1, 0.05$  and  $0.001$  to see that it really imitates the Dirac-Delta function. Hence, obtain the value of the integral and print the answer.

$$\int_0^{\pi} (x^3 - 4x + 2) G_{\sigma}(x - x_0) dx.$$

[Code : 10 + Plot : 2, Total : 12]

- (b) Write a Python program using SciPy module to calculate the following Gaussian integral :

$$I = \int_0^{\infty} e^{-t^2/2} dt$$

Plot the integrand up to an upper limit of your choice. Print the value of the integral.

[Code : 6 + Plot : 2, Total : 8]

5. (a) Charging of a CR circuit is described by the differential equation :

$$R \frac{dq}{dt} = \frac{CV_0 - q}{C},$$

where  $R$ ,  $C$  and  $V_0$  are the resistance, capacitance and battery voltage. Write a Python program using SciPy module to solve this equation and plot  $q(t)$  for  $0 \leq t \leq 5$ s. Take  $R = 1 \text{ k}\Omega$ ,  $C = 1 \text{ mF}$ ,  $V_0 = 10 \text{ V}$ .

Given, at  $t = 0$ ,  $q(0) = 0$ .

[Code : 8 + Plot : 2, Total : 10]

- (b) The Fourier expansion of a given function  $F(x)$  periodic over interval  $[-l, l]$  can be written as

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right) \right),$$

where  $a_n = \frac{1}{l} \int_{-l}^l F(x) \cos\left(\frac{n\pi x}{l}\right) dx, n = 0, 1, 2, 3, \dots, \infty$

$$b_n = \frac{1}{l} \int_{-l}^l F(x) \sin\left(\frac{n\pi x}{l}\right) dx, n = 1, 2, 3, \dots, \infty.$$

Write a program to compute the first ten Fourier's coefficients ( $a_n, b_n$ ) of a square wave. Plot the constructed series and the square wave signal in the same graph using matplotlib. Set appropriate axis labelling and legend at suitable location in the graph.

[Program : 6 + Graph : (1+1)+(1+1), Total : 10]

6. (a) Numerically verify the recursion relation of the given Legendre's polynomial.

$$(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$$

$x \in [-1, 1]$ . Take  $n = 1, 2, 3, 4$ . Plot the right hand side and left hand side of the given relation in the same graph.

[Program : 6 + Graph : 2, Total : 8]

- (b) Write a program to solve the 1D Diffusion equation :

$$\frac{\partial U(x,t)}{\partial t} = D \frac{\partial^2 U(x,t)}{\partial x^2}$$

$$D = 1.0$$

$$U(x,0) = \sin x, \quad 0 \leq x \leq \pi \quad \text{Initial Condition}$$

$$U(0,t) = U(\pi,t) = 0, \quad \forall t \quad \text{Boundary Condition}$$

Space Domain :  $x \in [0, \pi]$ , Time Domain :  $t \in [0, 2.0]$

Plot  $U(x, t)$  with respect to  $x$  for different time instants.

[Program : 10 + Graph : 2, Total : 12]

7. (a) Compute the integral

$$I = \int_1^{\infty} \frac{\ln x}{x^2} dx.$$

Plot the integrand up to certain upper limit of your choice.

[Program : 3 + Output : 1 + Graph : 2, Total : 6]

(b) Write a program to solve 1d wave equation :

$$\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial x^2}$$
$$U(0,t) = U(10,t) = 0, \quad t > 0$$
$$U(x,0) = e^{-k(x-x_0)^2}, \quad 0 \leq x \leq 10.$$
$$\left( \frac{\partial U}{\partial t} \right)_{x,t=0} = 0, \quad 0 \leq x \leq 10$$

Take :  $x_0 = 5.0$ ,  $k = 10.0$ . Plot the numerical  $U_N(x, t)$  solution with respect to  $x$  for different time instants ( $0 \leq t \leq 15$ ). Label the axes and put appropriate legends in the graph.

[Program : 10 + Graph : 2+(1+1), Total : 14]

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2023

PHYSICS — HONOURS — PRACTICAL

Paper : CC-8P

(Syllabus : 2019-2020 and 2018-2019)

Full Marks : 30

DAY - 3

[Experiment : 20, Laboratory Notebook : 5, Viva voce : 5]

1. (a) Hermite Polynomials satisfy orthogonality relation :

$$\int_{-\infty}^{\infty} H_n(x)H_m(x)e^{-x^2} dx = 2^n \cdot n! \sqrt{\pi} \delta_{nm}.$$

where,  $\delta_{nm} = 1$  for  $n = m$   
 $= 0$  for  $n \neq m$ .

Write a Python program using SciPy module to verify the above relation for  $n = m = 1$  and  $n = 1, m = 2$ . [Code : 8, Total : 8]

- (b) The heat equation in one dimension is given by :

$$K \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial u(x,t)}{\partial t}; K \equiv \text{constant},$$

where  $u(x, t)$  is the temperature function. Consider a wire of length  $0 \leq x \leq \pi$ .

The boundary conditions at both ends are :

$$u(0, t) = 0, \quad u(\pi, t) = 2\pi \text{ for } t > 0.$$

The initial temperature distribution is :

$$u(x, 0) = x + \sin x, \quad 0 \leq x \leq \pi.$$

Write a Python program to solve heat equation and plot the temperature distribution throughout the wire at  $t = 10$ s. Take  $K = 1.0$  unit.

[Code : 10 + Plot : 2, Total : 12]

2. (a) Write a Python program to show the convolution of the following Gaussian functions is also a Gaussian :

$$f(x) = \frac{1}{\sqrt{\pi}} e^{-(x+1)^2}, \quad g(x) = \frac{1}{2\sqrt{\pi}} e^{-\frac{(x-1)^2}{4}}$$

Plot the functions  $f(x)$ ,  $g(x)$  and the convoluted function on the same graph.

$$\text{Note: } (f * g)(t) := \int_{-\infty}^{\infty} f(\tau)g(t-\tau) d\tau$$

[Code : 10 + Plot : 2, Total : 12]

- (b) Solve

$$\frac{d^2x}{dt^2} + 4t = \sin^2 2t, \quad x(0) = 0, \quad \left. \frac{dx}{dt} \right|_0 = 0$$

by using odeint package from SciPy. Take  $t \in [0, 5]$

$$\text{Exact solution : } x(t) = -\frac{1}{96}(-3 + 24t^2 - 64t^3 + 3\cos 4t)$$

- (i) Plot  $x(t)$  vs.  $t$  and compare it with exact solution.

- (ii) Plot  $\frac{dx}{dt}$  vs.  $t$ .

[Program : 5 + Graph : 1+1+1, Total : 8]

3. (a) The number of surviving nuclei in a radioactive sample is given by :

$$\frac{dN(t)}{dt} = -\lambda N(t)$$

Write a Python program to solve this equation to find the number of nuclei present after  $t = 20$  years, if the sample had  $10^4$  nuclei at the beginning ( $t = 0$ ). Take  $\lambda = 6.93 \times 10^{-3} \text{ year}^{-1}$ . Print the answer.

Plot  $N(t)$  vs.  $t$  for  $t$  in  $\left[0, \frac{5}{\lambda}\right]$ .

[Code : 8 + Plot : 2, Total : 10]

- (b) The Fourier expansion of a given function  $F(x)$  periodic over interval  $[-l, l]$  can be written as

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right) \right),$$

where  $a_n = \frac{1}{l} \int_{-l}^l F(x) \cos\left(\frac{n\pi x}{l}\right) dx, n = 0, 1, 2, 3, \dots, \infty$

$$b_n = \frac{1}{l} \int_{-l}^l F(x) \sin\left(\frac{n\pi x}{l}\right) dx, n = 1, 2, 3, \dots, \infty.$$

Write a program to compute the first five Fourier's coefficients ( $a_n, b_n$ ) of a square wave. Plot the constructed series and the square wave signal in the same graph using matplotlib. Set appropriate axes labelling and legend at suitable location in the graph.

[Program : 6 + Graph : (1+1)+(1+1), Total : 10]

4. (a) Write a program to solve 1d wave equation :

$$\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial x^2}$$
$$U(0,t) = U(10,t) = 0, \quad t > 0$$
$$U(x,0) = e^{-k(x-x_0)^2}, \quad 0 \leq x \leq 10.$$
$$\left(\frac{\partial U}{\partial t}\right)_{x,t=0} = 0, \quad 0 \leq x \leq 10.$$

Take :  $x_0 = 5.0$ ,  $k = 10.0$ . Plot the numerical  $U_N(x, t)$  solution with respect to  $x$  for different time instant  $t(0 \leq t \leq 20)$ . Label the axes and put appropriate legends in the graph.

[Program : 10 + Graph : 2+(1+1), Total : 14]

(b) Compute the integral

$$\int_1^2 \frac{x dx}{\sqrt{x-1}}$$

Plot the integrand in the range  $x \in [1.001, 2.0]$ . Label the axes and write the integrand as a legend inside the graph.

[Program : 3 + Output : 1 + Graph : 1+1, Total : 6]

5. (a) Write a program to solve Laplace's equation on square grid

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

Dirichlet Boundary conditions are :

Left B. C.  $U(0, y) = \frac{y}{1+y^2}$  for  $0 < y < 1$

Right B. C.  $U(1, y) = \frac{y}{4+y^2}$  for  $0 < y < 1$

Bottom B. C.  $U(x, 0) = 0.0$  for  $0 < x < 1$

Top B. C.  $U(x, 1) = \frac{1}{(1+x)^2 + 1}$  for  $0 < x < 1$

The exact analytical solution is  $\frac{y}{(1+x)^2 + y^2}$ . Plot the numerical solution ( $U_N$ ) and exact analytical solution ( $U_A$ ) with respect to  $x$  and  $y$  in the same graph.

[Program : 8 + Graph : 2+2, Total : 12]

- (b) Verify the orthogonality relation :

$$\int_{-1}^1 P_n(x)P_m(x)dx = \frac{2}{2n+1} \delta_{mn}, \text{ for } m, n = 1, 2, 3, 4,$$

where  $P_n$  is the  $n$ th order Legendre's polynomial.

[Program : 8, Total : 8]

6. (a) Dirac-Delta function can be expressed as the limit of Gaussian distribution :

$$\delta(x-x_0) = \lim_{\sigma \rightarrow 0} G_{\sigma}(x-x_0),$$

$$\text{where, } G_{\sigma}(x-x_0) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-x_0)^2}{2\sigma^2}\right\}.$$

Take  $x_0 = 0$ . Plot  $G_{\sigma}(x-x_0)$  for  $\sigma = 1.0, 0.5$  and  $0.005$  to see that it really imitates the Dirac-Delta function. Hence, obtain the value of the following integral :

$$\int_{-1}^1 e^{x+2} G_{\sigma}(x-x_0) dx.$$

Print the answer.

[Code : 9 + Plot : 2, Total : 11]

- (b) Suppose  $f(t) = e^{-at^2}$  and  $g(t) = e^{-bt^2}$ , are two Gaussians, where  $a, b > 0$ . Taking suitable values of  $a$  and  $b$ , show that the convolution of these two Gaussians is also a Gaussian. Plot them.

$$\text{Note : } (f * g)(t) := \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau.$$

[Program : 7 + Graph : 2, Total : 9]

7. (a) Consider the following forced harmonic motion :

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + kx = F \cos \omega t,$$

where the exact solution of the amplitude of the steady state,  $A = \frac{F}{\sqrt{(k - \omega^2)^2 + \gamma^2 \omega^2}}$ . Take  $\gamma = 0.1$ ,

$k = 2$  and  $F = 1$ . Solve the differential equation by using odeint from SciPy module. Plot the amplitude resonance curve. Compare with the exact formula.

[Program : 7 + Graph : 2+1, Total : 10]

- (b) Compute the improper integral

$$\int_a^b dx \frac{1}{\sqrt{(b-x)(x-a)}}$$

Take the following combinations of  $a$  and  $b$  :

$$a = 1, b = 2$$

$$a = 2, b = 4$$

Plot the integrand vs.  $x$  for any set of  $(a, b)$ .

[Program : 6 + Output : 1+1 + Plot : 2, Total : 10]

**2023**

**PHYSICS — HONOURS — PRACTICAL**

**Paper : CC-8P**

**(Syllabus : 2019-2020 and 2018-2019)**

**Full Marks : 30**

**DAY - 4**

**[Experiment : 20, Laboratory Notebook : 5, Viva voce : 5]**

1. (a) Consider a rod (length  $L$ ) of uniform cross-section and made of homogeneous material which is perfectly insulated so that heat flows only along the length. Two ends of the rod are inserted into two ice baths to maintain fixed temperature at both the ends and  $U\left(\frac{L}{2}, 0\right) = 100$ . Consider the following 1D diffusion equation :

$$\frac{\partial U(x,t)}{\partial t} = \frac{\partial^2 U(x,t)}{\partial x^2}$$

Using one initial condition and two boundary conditions, solve this partial differential equation and plot the temperature profile along the length of the rod with time.

[Program : 10 + Graph : 2, Total : 12]

- (b) Evaluate the Integral for  $n = -3/2, -1/2, 1/2, 1, 3/2, 5/2$ .

$$I(n) = \int_0^{\infty} e^{-x} x^{n-1} dx.$$

[Program : 4 + Output : 4, Total : 8]

2. (a) Suppose  $f(t) = e^{-at}$  and  $g(t) = \sin(bt)$ . Taking suitable values of  $a$  and  $b$ , compute the convolution of the given functions. Plot  $f(t)$ ,  $g(t)$  and convoluted function.

Note : Convolution over finite range :  $(f * g)(t) := \int_0^t f(\tau)g(t-\tau)d\tau$ .

[Program : 8 + Graph : 2, Total : 10]

- (b) Solve

$$\frac{d^2x}{dt^2} + Ax + Bx^3 = C \cos \omega t$$

by using odeint package from SciPy. Take :  $A = 1.2$ ,  $B = 0.2$ ,  $C = 1.8$  and  $\omega = 1.5$ ,  $t \in [0, 15]$

- (i) Plot  $x$  vs.  $t$ . (ii) Plot  $\frac{dx}{dt}$  vs.  $t$ .

Label the axes and put appropriate legends in the graph.

[Program : 7 + Graph : 1+1+1, Total : 10]

3. (a) The Fourier expansion of a given function  $F(x)$  periodic over interval  $[-l, l]$  can be written as

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi x}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right) \right),$$

where  $a_n = \frac{1}{l} \int_{-l}^l F(x) \cos\left(\frac{n\pi x}{l}\right) dx, n = 0, 1, 2, 3, \dots, \infty$

$$b_n = \frac{1}{l} \int_{-l}^l F(x) \sin\left(\frac{n\pi x}{l}\right) dx, n = 1, 2, 3, \dots, \infty.$$

Write a program to compute the first five Fourier's coefficients ( $a_n, b_n$ ) of a square wave. Plot the constructed series and the square wave signal in the same graph using matplotlib. Set appropriate axes labelling and legend at suitable location in the graph.

[Program : 6 + Graph : (1+1)+(1+1), Total : 10]

(b) An useful improper integral is :

$$\int_0^{\infty} x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}$$

(i) Using SciPy module, show the result for  $n = 3, \alpha = 1$  :

$$\int_0^{\infty} x^3 e^{-x} dx = 6$$

(ii) Such integrals are usually performed numerically by gradually increasing the upper limit to

a large value (say,  $10^3$ ). Plot the value of the integral :  $I(a) = \int_0^a x^3 e^{-x} dx$  against the upper

limit 'a'. The plot must converge to the desired answer.

[Code : 3+5+ Plot : 2, Total : 10]

4. (a) Write a program to solve Laplace's equation on square grid

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0$$

Dirichlet Boundary conditions are :

Left B. C.  $U(0, y) = 0.0$  for  $0 < y < 1$

Right B. C.  $U(1, y) = y$  for  $0 < y < 1$

Bottom B. C.  $U(x, 0) = 0.0$  for  $0 < x < 1$

Top B. C.  $U(x, 1) = x$  for  $0 < x < 1$

The exact analytical solution is  $U_A(x, y) = xy$ . Plot the numerical solution ( $U_N$ ) and exact analytical solution ( $U_A$ ) with respect to  $x$  and  $y$  in the same graph.

[Program : 10 + Graph : 1+1, Total : 12]

- (b) Verify the relation :

$$\int_{-\infty}^{\infty} \delta(x-a)f(x)dx = f(a)$$

Limiting representation of Dirac-Delta function :  $\delta_n(x-a) = \lim_{n \rightarrow \infty} \sqrt{\frac{n}{\pi}} e^{-n(x-a)^2}$ .

Take,  $f(x) = x^2$  and  $a = 2$ . Plot  $\delta_n(x-a)$  vs.  $x$  for four values of  $n$ . Set appropriate axes labelling and put legend in the graph.

[Program : 6 + Graph : 2, Total : 8]

5. (a) Wave equation in 1D is given by

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}, \quad c = \text{velocity.}$$

Consider  $c = 1$ ,  $0 \leq x \leq \pi$  and  $0 \leq t \leq 10$ s. The initial conditions are :

$$y(x, 0) = \sin(x), \quad \frac{\partial y(x, 0)}{\partial t} = 0$$

The boundary conditions are :

$$y(0, t) = 0 = y(\pi, t).$$

Write a suitable Python program to solve this differential equation. Print  $y\left(\frac{\pi}{2}, 5\right)$ .

Plot  $y(x, t)$  vs.  $x$  for a given  $t$ .

[Code : 10 + Plot : 2, Total : 12]

(b) Hermite Polynomials satisfy orthogonality relation :

$$\int_{-\infty}^{\infty} H_n(x) H_m(x) e^{-x^2} dx = 2^n \cdot n! \sqrt{\pi} \delta_{nm}$$

where,  $\delta_{nm} = 1$  for  $n = m$   
 $= 0$  for  $n \neq m$

Write a Python program using SciPy module to verify the above relation for  $n = m = 1$  and  $n = 1$ ,  $m = 2$ .

[Code : 8, Total : 8]

6. (a) Solve  $\frac{d^2x}{dt^2} - \epsilon(1-x^2)\frac{dx}{dt} + x = 0$ ,  $x(0) = 0.75$ ,  $\frac{dx}{dt}(0) = 0$  by odeint package from SciPy.

Take  $\epsilon = 3.5$ .

(i) Plot  $x(t)$  vs.  $t$  and  $\frac{dx}{dt}$  vs.  $t$  in the same graph using matplotlib. Label the x-axis as 'Time' and y-axis as 'Displacement or Velocity'. Set legends at the appropriate position for two different graphs.

(ii) Plot  $\frac{dx}{dt}$  vs.  $x(t)$ . Label the x-axis as 'Position' and y-axis as 'Velocity'. Set title as 'Phase plot'.

[Program : 6 + Graph : (2+2)+(1+1), Total : 12]

(b) Write a python program to numerically evaluate the Bose-Einstein integral :

$$\int_0^{\infty} \frac{x}{e^x - 1} dx = \frac{\pi^2}{6}$$

Plot the integrand for  $0 \leq x \leq 10$ . Start with a small non-zero value as the lower limit.

[Code : 6 + Plot : 2, Total : 8]

7. (a) Using SciPy module, write a Python program to solve the differential equation describing the motion of a particle under gravity subjected to a resistive drag force :

$$\frac{dv}{dt} = g - kv, \text{ where } v = \frac{dx}{dt},$$

'x' being the displacement of the particle.

Consider  $k = 0.01$  unit and  $v(t = 0) = 0$ . Plot  $v(t)$  vs.  $t$  for  $0 \leq t \leq 10$ s.

[Code : 10 + Plot : 2, Total : 12]

- (b) Numerically verify the recursion relation of the given Legendre's polynomial.

$$(1-x^2)P'_n(x) = (n+1)xP_n(x) - (n+1)P_{n+1}(x)$$

$x \in [-1, 1]$ . Take  $n = 2, 4$ . Plot the right hand side and left hand side of the given relation in the same graph.

[Program : 5 + Graph : 3, Total : 8]

2022

PHYSICS — HONOURS — PRACTICAL

Paper : CC-8P

[Mathematical Physics-II]

(Syllabus : 2019-2020)

Full Marks : 30

*The figures in the margin indicate full marks.*

[Distribution of Marks : LNB - 5, Viva - 5, Experiment - 20, Total - 30]

1. (a) Consider the following Gaussian integral :

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-(x-\mu)^2/2\sigma^2} dx$$

Take suitable values of  $\sigma$  and  $\mu$  and plot the function. Find the value of the integral.

- (b) Suppose  $F(t) = -at^2$  and  $G(t) = -bt^2$ , are two Gaussians, where  $a, b > 0$ . Taking suitable values of  $a$  and  $b$ , show that the convolution of these two Gaussians is also a Gaussian. Plot them.

8+12

2. (a) Consider the following damped harmonic motion :

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + kx = 0$$

Take  $\gamma = 0.4$  and  $k = 1$ . Solve the differential equation by using a suitable function from scipy module and plot the decay of  $x$  versus  $t$  for a definite range.

- (b) Consider the Wave functions for 1D quantum harmonic oscillator :

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!}} \left(\frac{1}{\pi}\right)^{\frac{1}{4}} \exp\left(-\frac{x^2}{2}\right) H_n(x)$$

where  $H_n$  is the Hermite polynomial and we consider  $\frac{m\omega}{h} = 1$ . Plot the wave functions for  $n = 0, 1$  and  $2$  in a single graph.

12+8

( 2 )

3. (a) Consider the following forced harmonic motion :

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + kx = F \cos \omega t$$

where the exact solution of the amplitude of the steady state =  $F / \sqrt{(k - \omega)^2 + \gamma^2 \omega^2}$ . Take  $\gamma = 0.5$ ,  $k = 5$  and  $F = 1$ . Solve the differential equation by using a suitable function from scipy module and plot the amplitude resonance curve. Compare with the exact formula.

- (b) Prove the orthogonality relation :

$$\int_{-1}^1 P_m(x) P_n(x) dx = \frac{2}{2n+1} \delta_{mn}$$

where  $P_m$  is the  $m$ -th order Legendre polynomial.

12+8

4. (a) Analyse in a Fourier series of a Sawtooth wave whose window size is 6 and frequency is 2. Evaluate the Fourier coefficients and plot the constructed series with the given periodic function.

- (b) Consider a rod (length L) of uniform cross-section and made of homogeneous material which is perfectly insulated so that heat flows only along the length. Two ends of the rod are inserted in two ice baths to maintain fixed temperature at both the ends and heat at the middle. Consider the following diffusion equation :

$$\frac{\partial u}{\partial t} = D^2 \frac{\partial^2 u}{\partial x^2}$$

where  $D$  is the diffusion coefficient and may be set to unity. Forming one initial condition and two boundary conditions, solve this partial differential equation and plot the temperature profile (temperature along the length of the rod).

8+12

5. (a) Evaluate the Fresnel Integral :

$$\int_0^{\infty} \sin x^2 dx$$

Plot the solution up to a certain value of the upper limit. Comment on the plot.

- (b) Write a python program to analyse in a Fourier series of a full-wave rectifier signal :

$$f(x) = |\sin(x)|$$

Evaluate the Fourier coefficients and plot the constructed series with the given periodic function.

10+10

3. (a) Consider the following radioactive decay equation :

$$\frac{dx}{dt} = -kx$$

Solve the differential equation by using a suitable function from scipy module and plot decay of  $x$  versus  $t$ .

- (b) Prove the orthogonality relation :

$$\int_{-1}^1 P_m(x)P_n(x)dx = \frac{2}{2n+1} \delta_{mn}$$

where  $P_m$  is the  $m$ -th order Legendre polynomial.

12+8

4. (a) Analyse in a Fourier series of a square wave whose window size is 6 and frequency is 2. Evaluate the Fourier coefficients and plot the constructed series with the given periodic function.
- (b) Consider a rod (length  $L$ ) of uniform cross-section and made of homogeneous material which is perfectly insulated so that heat flows only along the length. The right end of the rod is inserted in an ice bath to maintain fixed temperature at that end and heated at the left end. Consider the following diffusion equation :

$$\frac{\partial u}{\partial t} = D^2 \frac{\partial^2 u}{\partial x^2}$$

where  $D$  is the diffusion coefficient and may be set to unity. Forming one initial condition and two boundary conditions, solve this partial differential equation and plot the temperature profile (temperature along the length of the rod).

8+12

5. (a) Evaluate the Fresnel Integral :

$$\int_0^{\infty} \sin x^2 dx$$

Plot the solution up to a certain value of the upper limit. Comment on the plot.

- (b) Write a python program to analyse in a Fourier series of a symmetric triangular signal. Evaluate the Fourier coefficients and plot the constructed series with the given periodic function. 10+10

6. (a) Write a suitable python program to calculate the integral :  $\int_0^{\infty} e^{-x^2} dx$ . Set the accuracy level to 0.0001.

- (b) Write a suitable program to solve the following equation :

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \text{ where } 0 < x < \pi, t > 0 \text{ with the initial and boundary conditions :}$$

$$u(x, 0) = 1, \frac{\partial u(x, 0)}{\partial t} = 0$$

$$u(0, t) = 0, u(\pi, t) = 0.$$

8+12

7. (a) Verify the following identity with a python program :

$$\frac{\sin(n+1)\theta}{\sin\theta} = \sum_{l=0}^n P_l(\cos\theta) P_{n-l}(\cos\theta), \text{ where the symbols have their usual meanings.}$$

- (b) Consider the Cauchy problem :

$$\frac{\partial u}{\partial t} = x \frac{\partial u}{\partial x} - u + 1, -\infty < x < \infty, t \geq 0.$$

$$u(x, 0) = \sin x, -\infty < x < \infty$$

Solve this problem and discuss the behaviour of the solution for large time.

10+10