51. In an experiment using a spectrometer in normal adjustment fitted with a plane transmission grating and using monochromatic light of wavelength 5.89×10^{-10} cm, diffraction maxima were obtained with telescope setting at 153°44', 124°5', 76°55' and 47°16', the central maxima being at 100°30'. Show that these observations are consistent with normal incidence. Calculate the number of rulings per cm. of the grating. [Ans. 6800]

52. A diffraction grating has 4000 lines per cm. Calculate the dispersive power of the grating in the 3rd order spectrum when light of wavelength 5000 A° is incident on it normally. [Ans. 15000]

4

POLARISATION OF LIGHT

❒ 4.1. Introduction :

Huygen's wave theory fully explains the phenomena of interference and diffraction. This establishes the wave nature of light. But we are aware of two kinds of wave *viz.*, (i) transverse and (ii) longitudinal. What kind of wave light is? Interference and diffraction do not give us an answer to this important question because both types of wave exhibit the two phenomena. It was, however, thought for a long time after the wave theory was revived that the vibrations of light occurred in the same direction as the light wave travelled, analogous to sound waves. Thus light waves were thought to be longitudinal waves. But subsequent observations and experiments—specially those on polarisation of light, showed that the vibrations of light occur in planes *perpendicular* to the direction along which light wave travels and thus proved that light waves are *transverse waves*.

It was Huygens who first noticed the phenomenon of polarisation in 1690. Observing the behaviour of a beam of light which had been transmitted through a piece of icelandspar crystal, he noticed this peculiar property of light. In subsequent articles, we will see that polarisation is possible only in the case of a transverse wave. For this reason, light wave is regarded as transverse wave. In this connection it may be said that sound in air is also propagated as a wave; but as sound waves do not exhibit polarisation, they are not transverse; they are, in fact, longitudinal waves.

❒ 4.2. Polarised light ; Experiment with tourmaline crystal :

In chemical composition, tourmaline is made up of silicates of various metals and boron. It is a natural crystalline mineral having some interesting optical and other effects. The crystal is hexagonal in shape, almost transparent with a slight greenish tinge. The greatest diagonal of a section of the crystal is called its *optic axis* (*ab* in fig. 4.1).

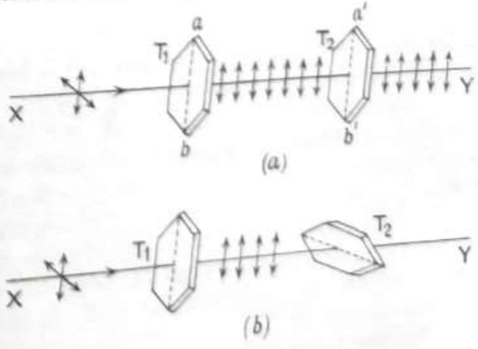

Fig. 4.1

In fig. 4.1 T_1 is a thin piece of tourmaline crystal. A fine beam of light moving along the direction *XY* is incident normally on the flat surface of the crystal. Some of the light will be transmitted through the crystal. There will be no change in the intensity of the emergent light except that it has a slight tinge, the colour depending upon the nature of the crystal. If the crystal T_1 is now rotated about the line XY as axis, there will still be no change in the intensity of the transmitted beam.

Now, the transmitted light is allowed to fall on another similar crystal T_2 placed behind the first and the light passing through T_1 and T_2 is observed. When the crystals have their axes ab and $a'b'$ parallel, the light passing through T_2 appears slightly darker but without any change in intensity. If T_2 is now rotated slowly about the line of vision with its plane parallel to T_1, the light passing through T_2 becomes dimmer and disappears completely when the axes ab and $a'b'$ are perpendicular to each other. In this case, the crystals are said to be in *crossed position*. When T_2 is rotated further, the light reappears and becomes brightest when the axes ab and $a'b'$ are again parallel.

Same result will be obtained if the crystal T_1 is rotated, and the crystal T_2 is kept fixed.

This simple experiment leads to the conclusion that light waves are transverse waves; otherwise the light emerging from T_2 could never be extinguished by simply rotating the crystal T_2. To understand it clearly we shall take an analogous mechanical illustration.

Consider a horizontal rope CD attached to a fixed point D at one end [fig. 4.2]. Transverse waves due to vibration in many different planes can be set up along the string by holding the end C in the hand and moving it up and down in all directions perpendicular to CD. Suppose we repeat the experiment but this time we have two parallel slits S_1 and S_2 between C and D, as shown in the diagram. A wave then emerges along the line S_1S_2, but unlike the waves along the part CS_1 of the rope, which are due to vibrations in many different planes, the wave along the line S_1S_2 is due only to vibrations parallel to the slit S_1. *Such waves, containing vibrations in a given plane, are called polarised waves.* The polarised wave passes through the parallel slit S_2. But when S_2 is turned so that it is *perpendicular* to S_1 as shown in Fig. 4.2(b), no wave is obtained beyond S_2.

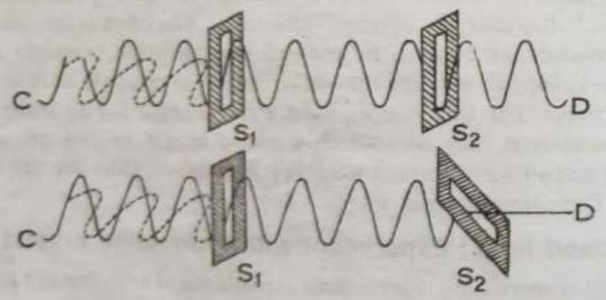

Fig. 4.2

If the rope CD is replaced by a thick elastic cord and *longitudinal waves* are produced along CD, then turning the slit S_2 round from the position shown in fig. 4.2(a), to that shown in fig. 4.2(b) makes no difference to the wave—it travels through S_1 and S_2 undisturbed.

Now let us go back to the experiment of tourmaline crystal described a little while ago. The fact that light coming out of the crystal T_2 can be completely cut off in a particular position of the crystal shows that light waves are of transverse nature. Had the light waves been longitudinal, they could not have been stopped in any position of the crystal T_2. So ordinary light may be regarded as transverse waves due to vibrations of the particles of the medium in many different directions in a plane perpendicular to the direction of propagation [fig. 4.3]. This beam of light is called ordinary or *unpolarised beam of light. When such unpolarised beam of light has passed through the crystal* T_1, the crystal because of its internal molecular structure, has transmitted only those vibrations of light which are parallel to its optic axis ab [fig. 4.1(a)]. Consequently the transmitted beam contains vibrations which are confined in a particular direction in a plane perpendicular to the direction of propagation. Such one-sided property of light is known as *polarisation* and the beam is called a *polarised beam.* The polarised light obtained beyond the crystal T_1 can pass through the second crystal T_2 only when the optic axis $a'\ b'$ of the second crystal is parallel to the vibrations of the polarised beam and hence parallel to the optic axis ab of the first crystal T_1. When the crystals are in crossed position, no light emerges beyond T_2 because the vibrations of the polarised beam are at right angles to the axis $a'b'$ of the crystal T_2.

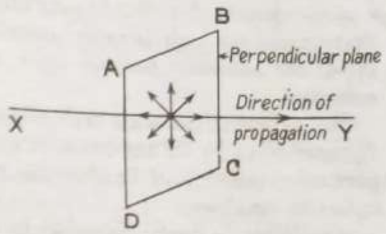

Fig. 4.3

When the axes of the crystals are inclined to each other, some of the polarised light emerges from the crystal T_2 and the remaining part is absorbed. Consequently the intensity of the emergent light diminishes. Suppose XY is the direction of propagation of light and ab is the optic axis of the first crystal T_1 [Fig. 4.4]. On emergence from the crystal T_1, the vibrations of the polarised beam will be confined only in the direction ab. If the axis $a'b'$ of the second crystal T_2 is kept inclined at an angle θ with the axis ab of the first crystal (it is to be brone in mind that the planes of the crystals are always parallel; only the axes turn round), the second crystal does not allow all the light to come out of it; only the component of the vibrations of the polarised beam along $a'b'$ *i.e.*, $ab \cos\theta$ will be allowed transmission. So the intensity of the emergent light is proportional to $\cos\theta$. When $\theta = 90°$ *i.e.*, when the crystals are crossed, $\cos\theta = 0$ and no light emerges from the second crystal.

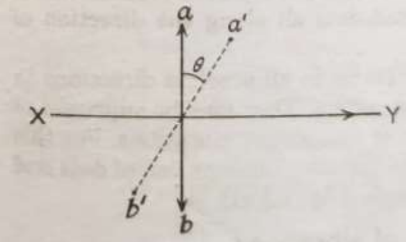

Fig. 4.4

The results of the above experiment with tourmaline crystals may be summarised as follows :

(i) Light is a transverse wave; it may be imagined as due to vibrations which occur in every one of the millions of planes which pass through the direction of propagations of light and perpendicular to it. This light is known as ordinary or unpolarised light.

(ii) When unpolarised light falls on a tourmaline crystals, it absorbs light due to vibrations in a particular direction and allows light due to vibrations in a perpendicular direction to pass through. For this one–sided property, the light that passes through is called polarised light. In fig. 4.1(a) the cryastal T_1 is called *the polariser* because light in passing through this crystals has been polarised.

(iii) Whether the beam is polarised or not may be tested with another crystal T_2 similar to T_1. If the beam is polarised, it will be completely cut off at a particular position of T_2. For this reason, the second crystal in fig. 4.1(a) is called *an analyser.*

(iv) When a beam polarised by a polariser is completely cut off by the analyser, the polariser and the analyser are said to be in crossed positions.

☐ 4.3. Conventions for drawing unpolarised and polarised light :

If, in a beam of light, vibrations take place in a given direction in a plane perpendicular to the direction of propagation of light, the beam is called a polarised beam ; if on the other hand, the vibrations take place in all possible directions, the beam is called an unpolarised beam. A convention is followed to represent them in the plane of the paper. The convention is as follows :

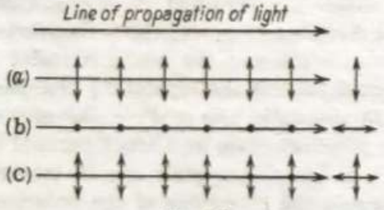

Fig. 4.5

(i) If the vibrations of the polarised beam take place *in the plane* of the paper, they are represented by small straight lines perpendicular to the direction of propagation with opposite arrow heads [fig. 4.5(a)].

(ii) If the vibrations of the polarised beam take place *perpendicular* to the plane of paper, they are represented by small dots all along the direction of propagation of light. [fig. 4.5(b)].

(iii) An unpolarised beam consists of vibrations in all possible directions in a plane perpendicular to the direction of propagation. They may be supposed to be the resultant of two mutually perpendicular component vibrations. For this reason an unpolarised beam is represented by the simultaneous use of dots and small straight lines with opposite arrow heads. [fig. 4.5(c)].

☐ 4.4. Plane of polarisation and plane of vibration :

The plane in which the vibrations of a polarised beam are confined is called *the plane of vibration.* In fig. 4.6, a polarised beam transmitted through a tourmaline crystal *AB* has been shown. A plane *ABCD* containing the vibrations of the polarised beam and passing through *OG*, the direction of propagation of the beam, is called the plane of vibration.

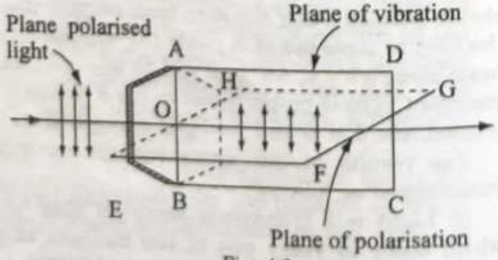

Fig. 4.6

A plane at right angles to the plane of vibration is known as *the plane of polarisation.* In fig. 4.6 *EFGH* represents the plane of polarisation. The plane of polarisation is thus a plane in which no vibrations occur.

☐ 4.5. Different kinds of polarised light and methods of their production :

In a polarised beam, the particles of the medium (*i.e.*, ether particles) are constrained to move in a particular path. In the case of the passage of light through a tourmaline crystal, the constrained path is a *linear* path. Such light is referred to as *plane-polarised* or *linearly polarised* light [Fig. 4.7(b)].

Under suitable conditions, however, the constrained path may be thought of as *circular* or *elliptical* path of fixed orientation transverse to the direction of propagation of the wave. We call them *circularly* or *elliptically polarised light* [Fig. 4.7(c) and (b)].

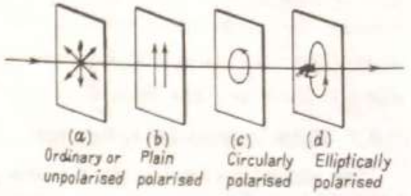
(a) Ordinary or unpolarised (b) Plain polarised (c) Circularly polarised (d) Elliptically polarised

Fig. 4.7

The following methods may be adopted to produce a plane—polarised light : (i) by reflection, (ii) by refraction, (iii) by selective absorption, (iv) by double refraction and (v) by scattering.

It may be proved that two plane polarised beams with suitable phase difference lying between 0° and 180°, when superposed may produce circularly or elliptically polarised beams.

☐ 4.6. Light waves are electromagnetic waves :

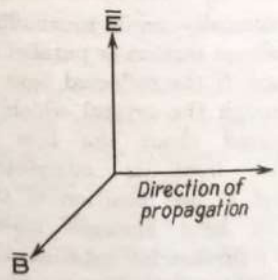

Fig. 4.8

Theory and experiment show that the vibrations of light are *electromagnetic* in origin ; a varying electric vector $\vec{E}$ and a varying magnetic vector $\vec{B}$ which has the same frequency and phase are present. $\vec{E}$ and $\vec{B}$ are perpendicular to each other and are in a plane at right angles to the direction of propagation of light (Fig. 4.8). Experiments have shown that the electric force in a light wave affects a photographic plate and causes fluorescence, while the magnetic force, though present, plays no part in this effect of a light wave. On this account of vibrations of the electric vector $\vec{E}$ are now chosen as the 'vibrations of light'. Thus we consider that the planes containing the light vibrations are those in which electric forces are present. In this context, a plane—polarised beam is one in which the vibrations of the electric vector are confined to a single direction.

Having thus identified the electric vector with the light vector, we can easily redefine the terms used in art. 4.4 in terms of the electromagnetic theory. The *plane of vibration* is thus the plane containing the electric vector $\vec{E}$ and the direction of propagation, the *plane of polarisation* being, as before, perpendicular to this plane.

A wave of light is said to be *linearly or plane—polarised* when the vibrations of $\vec{E}$ vector are parallel to each other for the wave *i.e.*, when all planes containing the vibrating $\vec{E}$ vector at different points of the wave and the direction of propagation of the wave are parallel to each other. An instantaneous picture of a plane-polarised wave, showing the vector $\vec{E}$ and $\vec{B}$ along a given ray has been shown in fig. 4.9.

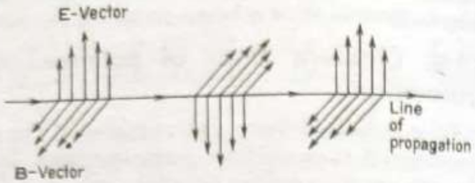

Fig. 4.9

❐ 4.7. Polarisation by reflection :

The easiest method to get a plane–polarised beam of light is reflection. In 1808 Malus discovered that a partially polarised light is obtained when ordinary light is reflected by a transparent reflecting surface, like a plane sheet of glass or a plane surface of water. He also found that the degree of polarisation depends upon the angle of incidence. Further, at a particular angle which depends on the nature of the reflecting surface, the reflected beam becomes totally polarised. This angle of incidence is known as the *angle of polarisation* of the reflecting surface. Experiment shows that the angle of polarisation for glass is about $57\frac{1°}{2}$.

Experiment : *AB* is the plane surface of a glass block, perpendicular to the plane of the paper [fig. 4.10]. A beam of unpolarised light *PO* is incident on the glass surface at an angle of incidence of $57\frac{1°}{2}$. The reflected light *OT* falls normally on a tourmaline crystal *T* whose section is parallel to its optic axis. If the reflected light is viewed through the crystal which is slowly rotated about the line of vision, the light is completely extinguished at one position of the crystal. The light emerges slowly again when the crystal is rotated further and after a further 90° rotation, the light emerges in its full strength. This proves that light reflected by the glass surface *AB* is plane–polarised.

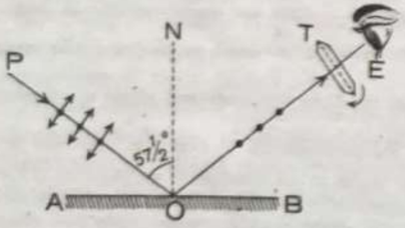

Fig. 4.10

It may be pointed out here that the state of polarisation of the reflected light can be analysed by another similar reflecting surface like *AB* instead of the tourmaline crystal (See next article).

Explanation : The production of polarised light by glass is explained as follows. Each of the vibrations of the incident (unpolarised) light can be resolved into a component parallel to the glass surface and a component perpendicular to the surface. According to the convention, the first component is represented by dot and the second one by short lines with opposite arrow heads. Now, the light due to the components parallel to the glass surface is largely reflected, but the remainder of the light, due mainly to the components perpendicular to the glass surface is refracted into the glass. Had the vibrations perpendicular to glass surface been carried by reflected beam, that would have given rise to longitudinal waves which do not exist for light. Thus, the light reflected by the glass is plane–polarised in the plane of the paper.

If the tourmaline crystal *T* is so placed that its optic axis lies in the plane of the paper, the vibrations of the reflected beam will be perpendicular to the optic axis and the crystal will cut off the light. On the other hand, if the crystal is so positioned that its optic axis is perpendicular to the plane of the paper, vibrations of the reflected beam are parallel to the optic axis. In this case the crystal transmits the light in its full strength. In other positions of the optic axis, light will be partially transmitted.

❐ 4.8. Biot's polariscope :

In the preceding article, it has been pointed out that the state of polarisation of the reflected beam may be analysed with another similar reflecting surface like *AB* instead of the tourmaline crystal. Biot devised an apparatus, known as *polariscope* where he used two reflecting surfaces as polariser and analyser. Fig. 4.11 shows the actual appearence of such a polariscope.

It consists of a transparent glass plate *B* fixed to two vertical pillars in such a way that it can turn about a horizontal axis. When the plate is turned, the angle of incidence of a beam of light falling upon it changes. *C* is another glass plate painted back in order to avoid internal reflection. It is also capable of turning about a horizontal axis. The frame in which the plate *C* is fixed can turn round a vertical axis and its angle of rotation can be measured from a scale graduated on the platform *S*.

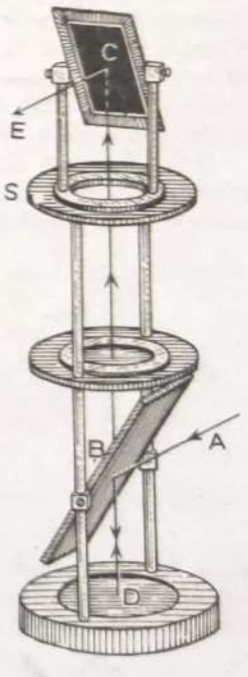

Fig. 4.11

When a ray of light is reflected by the plate B, it becomes partially polarised and falls upon a mirror *D* normally. Being reflected by the plane mirror *D*, the ray travels upward and is again reflected by the plate *C* in the direction *CE*. Now, to understand the functions of the plates *B* and *C* as polariser and analyser, consider the diagram 4.12.

Suppose, at first the plate *B* and *C* are parallel to each other [Fig. 4.12(a)]. In this condition, light will be incident on both the plates at equal angles of incidence. Now, the light reflected by *B* is plane–polarised and its vibrations will be perpendicular to the plane of the paper or parallel to the planes of the plates *B* and *C*. Consequently, the ray will be totally reflected by the plate *C*. If the frame containing the plate *C* be now rotated about the vertical axis so that the plane of *C* is perpendicular to that of *B* [Fig. 4.12(b)], no light will be reflected by the plate *C* because the vibrations of the polarised light are now perpendicular to the plane of *C*. It goes without saying that for complete obstruction of light by the plate *C*, the ray should be incident on the plate *B* at its polarising angle.

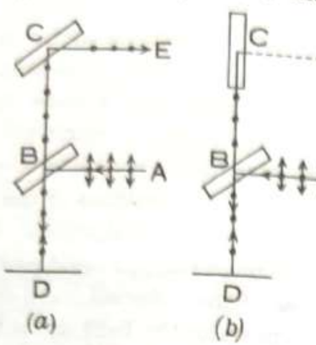

Fig. 4.12

□ 4.9. Law of Malus :

When a beam of light, polarised by reflection from a plane surface, is incident on another similar reflecting surface at the polarising angle, the intensity of twice reflected beam varies with the angle between the two surfaces. Thus, it was found in Biot's polariscope (art. 4.8 fig. 4.12) that the intensity of twice reflected beam is maximum when the two planes are parallel and minimum (*i.e.*, zero) when the two planes are perpendicular to each other. It is also applicable to twice transmitted beam from the polariser and analyser.

It was found by Malus *that the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polariser.* In the case of Biot's polariscope, this angle is evidently between the two reflecting planes. This is Malus' law.

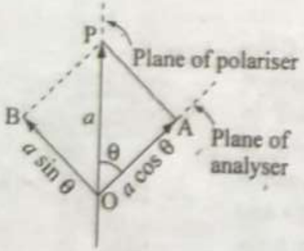

Fig. 4.13

Proof: Let us suppose that the angle between the planes of polariser and the analyser at any instant is θ. [Fig. 4.13]. Let $OP = a$ represent the amplitude of vibration of emergent plane–polarised light from the polariser. The amplitude a can be resolved into two components OA ($= a \cos \theta$) and OB ($= a \sin \theta$) along and perpendicular to the plane of transmission of the analyser. The parallel component OA is transmitted by the analyser while the perpendicular component OB is reflected. Thus, the intensity of the transmitted light through the analyser,

$$I_1 = (OA)^2 = (a \cos \theta)^2 = a^2 \cos^2 \theta = I.\ \cos^2 \theta$$

where I = the intensity of the plane–polarised light incident on the analyser. Thus, $I_1 \propto \cos^2 \theta$. This is Malus law.

The intensity of the transmitted light is maximum when the planes of analyser and the polariser are parallel, *i.e.*, when $\theta = 0$, $I_1 = I\ (\cos 0°)^2 = I$

Also the intensity of the emergent light is zero when the two planes are at right angles to each other (*i.e.*, polariser and the analyser are crossed) *i.e.*,

$$I_1 = I\ (\cos 90°)^2 = 0.$$

□ 4.10. Brewster's law :

A systematic study of the polarising angles of different media enabled Sir David Brewster to conclude that *the tangent of the polarising angle of a medium is numerically equal to the refractive index of the medium.* This is known as *Brewster's law.*

Mathematically, if p be the angle of polarisation for a medium whose refractive index is μ, then according to Brewster's law

$$\tan p = \mu$$

As μ of a medium varies with the colour of light, the angle of polarisation of a medium also varies with colour. This means that *white light cannot be completely plane–polarised by reflection.*

A very interesting result follows from Brewster's law. It may be shown in the following way that if a beam of light is incident on a reflecting medium at the polarising angle, the reflected and the refracted beams are at 90° to each other.

Let a ray of light AO be incident on a reflecting surface XY at the polarising $\angle p$ [Fig. 4.14]. The reflected ray OB, which is plane–polarised, also makes an $\angle p$ with the normal ON according to the laws of reflection. Let OC be the refracted ray. The angle of refraction is $\angle r$.

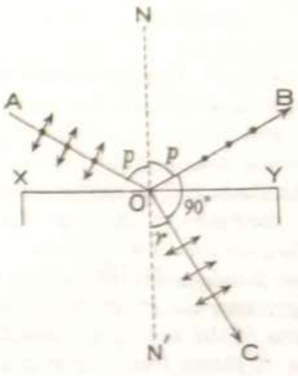

Fig. 4.14

According to Brewster's law, $\tan p = \mu$, where μ is the *r.i.* of the medium.

According to the laws of refraction $\mu = \dfrac{\sin p}{\sin r}$

$$\therefore \tan p = \frac{\sin p}{\sin r} \quad \text{or,} \quad \frac{\sin p}{\cos p} = \frac{\sin p}{\sin r}$$

or, $\sin r = \cos p = \sin (90° - p)$ or, $r = 90° - p$

or, $r + p = 90°$.

So, $\angle BOC = 90°$ *i.e.*, the reflected say OB and the refracted ray OC are at 90° to each other.

As the refractive index of a substance varies with the wavelength of the incident light, the polarising angle will be different for light of different wavelength. Polarisation will, therefore, be complete only of light of a particular wavelength at a time.

In the fig. 4.14 the incident ray AO is unpolarised, the reflected ray OB is completely polarised and the refracted ray OC is partially polarised. Their vibration have been represented by dots and short straight line with opposite arrow–heads.

□ 4.11. Polarisation by refraction ; pile of plates :

When light is incident at the polarising angle on the plane surface of a glass block, the reflected light is, no doubt, completely polarised but the intensity of the reflected light is very feeble because most of the light is refracted into the block. So, an intense polarised beam cannot be obtained by a single plate. Further, the emergent beam in a single plate, although very intense, is not completely polarised because, the reflected beam carries with it about 20% vibration in the perpendicular plane whereas the refracted beam is a mixture of 100% vibrations in the plane of the paper and 80% of the vibrations perpendicular to the plane of the paper. If instead of a single plate, a pile of plates is used and if unpolarised light is allowed to fall on the pile at the polarising angle, then multiple reflections will take place and at each reflection, the refracted beam will be robbed off 20% of its vibrations in the perpendicular plane. After a few reflections, fair percentage of plane–polarised light may be available. Finally two plane–polarised beams (one due to reflection whose vibrations are perpendicular to the plane of the paper and the other due to transmission whose vibrations are parallel to the plane of the paper) are completely separated from each other [Fig. 4.15]. The pile of plates thus provides us a simple method of producing plane–polarised light.

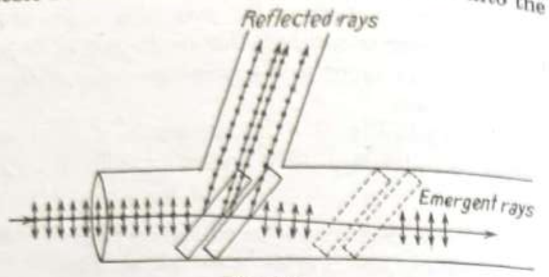

Fig. 4.15

If the emergent beam now falls on another similar pile held parallel to the first, light will come out of the second pile in full strength. On rotating the second pile about the line of incidence, the intensity of the emergent beam will gradaually diminish and will be completely dark when the two piles will be perpendicular to each other. So, the first pile may be called the polariser and the second pile the analyser.

The piles are generally mounted inclined in a tube so that ordinary (unpolarised) light is incident at the polarising angle. The transmitted light is then fairly plane polarised (fig. 4.14).

◻ 4.12. Polarisation by selective absorption :

If out of several wavelengths a substance absorbs one or a few of particular values, the absorption is called *selective absorption*. Almost all transparent coloured substances owe their colour to selective absorption because they absorb all the colours of the visible spectrum except one of its own colour which it transmits. A piece of red glass, for example, absorbs light of all colours except red which it transmits.

Several substances—specially tourmaline—produce plane-polarised beam by selective absorption. We have seen in art. 4.2 that when ordinary (unpolarised) light is transmitted through a thin piece of tourmaline crystal, the transmitted beam becomes plane–polarised with vibrations parallel to the optic axis of the crystal. As a matter of fact, ordinary light, on entering the crystal, is divided into two polarised components —one known as *ordinary ray* whose vibrations are perpendicular to the optic axis of the crystal and the other, known as *extra–ordinary ray* having vibrations parallel to the optic axis. The crystal absorbs the former component and transmits the latter. It is evident that thicker the crystal, greater is the absorption of the ordinary component. Hence, a thick piece of tourmaline crystal will transmit only the polarised extaordinary component of light. This is how a tourmaline crystal produces plane–polarised light by selective absorption.

Utilising the principle of selective absorption, *polarising films* of big size have been prepared. They are generally known as **polaroids.** Polarised beams may be produced very easily and at a low cost by polaroids. These films are made by setting small *herapathite* crystals on a nitro–cellulose film. Herapathite is an organic compound known as quinine iodosulphate.

Recently a still more efficient device called polariod–H has been prepared, which has made polarised light a thing of daily use. Thus goggles made of polaroid are more efficient in protecting the eyes from the glare of the sun than those prepared from coloured glass. These glasses do not allow light from shining surfaces to reach the eye. While acting as efficient shields for the eyes, they do not obscure the details of objects even in shade.

The head lights of a car are nowadays fitted with polaroids which allow vertical vibrations to pass through. The wind screens have polaroids in the crossed position. They do not allow light from head lamps of the cars coming from the opposite direction to pass to the eye and thus avoid glare. The visibility of road and the other objects is not materially affected since light scattered by them gets depolarised and can pass through the wind screens.

Polaroids have been used as polariser and analyser. They are also used as a good controller of light intensity for analysis of crystals and the study of their optical properties. Recently they have been used for stereoscopic picture projections in cinema.

◻ 4.13. Double refraction :

The phenomenon of double refraction was first observed by Bartholinus in 1669. He placed a crystal of icelandspar on some words written on a sheet of paper. To his surprise, he saw two images through the crystal. Bartholinus, therefore, gave the name of *double refraction* to the phenomenon. Experiments more than a century later showed that the crystal produced plane–polarised light when ordinary light was incident on it.

Usually when light travels through an *isotropic* medium, like water, glass etc., refraction takes place but light is not resolved into components because the optical properties in an isotropic medium, are the same in all directions. But there are some *anisotropic* crystals, such as calcite, quartz etc., in which the optical properties are different in different directions. Double refraction is a phenomenon found only in anisotropic medium.

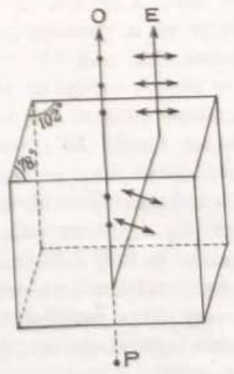

Fig. 4.16

Icelandspar is a crystalline form of calcite (calcium carbonate) which cleaves in the form of a 'rhomboid' when it is slightly tapped; this is a solid whose opposite faces are parallelograms. It is colourless and transparent. In fig. 4.16, an icelandspar crystal is placed over an ink dot (*P*) on a paper. Looking through the crystal, two images of the dot will be visible. If a glass block is placed instead of the calcite crystal, only one image will be seen. The two images seen through the calcite evidently represent two different emergent rays. This phenomenon of splitting of a ray into two during its passage through the calcite crystal is called double refraction. One of the rays is known as *ordinary ray, O* which obeys the laws of refraction while the other, known as *extraordinary ray, E* does not obey the laws of refraction. When the crystal is rotated about the line of vision, the image due to ordinary ray remains stationary but that due to extraordinary ray revolves round *O*. On account of this abnormal behaviour, the ray *E* is called extraordinary ray. Further it is seen that both the rays are plane–polarised with vibrations perpendicular to each other. Crystals like calcite which give rise to double refraction are called *birefringent*.

It is needless to mention that the refractive index (μ_0) of calcite or of any birefringent crystal for ordinary ray is constant but the refractive index (μ_e) for extraordinary ray, depending upon its direction of motion in the crystal, may be different. For example, μ_0 for calcite is 1.658 (for yellow ray) but μ_e varies from 1.468 to 1.658. For one direction, however, the refractive indices for ordinary and extraordinary rays are equal. This direction is called the **optic axis** of the crystal.

Experiments show that the image of *P* due to the ordinary ray appears closer to the observer than that due to extraordinary ray. This indicates that the refractive index of the crystal for ordinary ray is greater than that for the extraordinary ray, which again proves that the velocity of ordinary ray (V_0) in the crystal is less than the velocity of the extraordinary ray (V_e). Crystals of this type are known as *negative crystals*. Opposite things happen in quartz, tourmaline etc. In these crystals the velocity of ordinary ray is greater than the velocity of extraordinary ray. These crystals are, therefore, called *positive crystals*.

☐ 4.14. Some important definitions :

(a) *Optic axis* : In nature, calcite is available in various forms, but when slightly tapped, it cleaves in the form of a rhomb whose opposite faces are parallelogram. Fig. 4.17 shows a section of such a crystal. Each of the opposite faces is a parallelogram having angles about 102° and 78°. In all the corners of the rhomb except at *A* and *G* in fig. 4.18 surfaces meet at two acute angles and one obtuse angle. At corners *A* and *G* only, three faces meet at obtuse angles. They are called *blunt corners* of the crystal. If a straight line be drawn from *A* or *G* equally inclined to the three faces meeting at those points, then the straight line or any other straight line parallel to it is called the *optic axis* of the crystal. So, *the optic axis of a crystal is not a particular straight line ; it is rather a direction.* In fig. 4.16, *Aa* or *Ga* or any line parallel to it represents the optic axis. If the arms of the parallelogram are all equal, then the line joining *A* and *G* will be the optic axis [fig. 4.18].

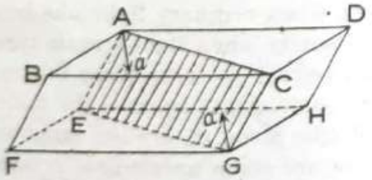

Fig. 4.17

The characteristic of optic axis is that when a ray is incident on a crystal parallel to its optic axis, the ray will not suffer double refraction inside the crystal *i.e.*, in this case, the ordinary and the extraordinary rays travel *in the same direction* and *with same velocity*. Some crystals like calcite, tourmaline, quartz etc., have only one optic axis. They are known as *uniaxial crystals*. Crystals like mica, borax etc., have two optic axes and therefore they are *biaxial crystals*.

(b) *Principal section* : If a section of the crystal be taken by a plane perpendicular to the opposite end faces and containing the optic axis, it will be called the *principal section* of the crystal. In fig. 4.18, *ACGE* is a principal section. It is perpendicular to the end faces *ABCD* and *EFGH* and it contains the optic axis *Aa* or *Ga*. As there are many straight lines parallel to the optic axis, so there will be many principal sections of a crystal. Evidently the principal section will be a parllelogram with its angles of about 109° (or 108°) and 71° (or 72°) [fig. 4.18(a)].

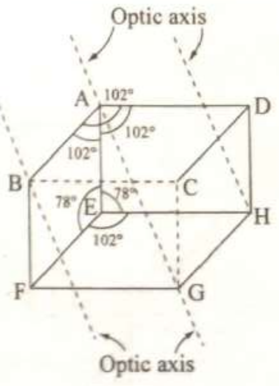

Fig. 4.18

(c) *Principal plane* : The principal plane with respect to a ray (either ordinary or extraordinary) means a plane which passes through the ray and the optic axis. For normal incidence, principal section coincides with the principal plane.

☐ 4.15. Polarisation by double refraction :

Consider a ray of light incident normally at the point *P* on the surface *ABCD* of a calcite crystal [Fig. 4.19(b)]. The principal section *ACGI* of the crystal has been shown separately in the fig. 4.19(a). As soon as the ray enters the rhomb of calcite, it will have double refraction, the ordinary ray *PO* emerging out of the crystal without any deviation [Fig. 4.19(a)]. The ordinary ray (*O*), as we know, obeys the laws of refraction. The other one—the *E*-ray —will suffer some deviation but as the upper and lower surfaces of a rhomb are parallel to each other, this ray will emerge from the crystal parallel to the incident direction *i.e.*, parallel to *PQ*. From the figure, it is apparent that there will be some lateral displacement of the ray. This extraordinary ray (*E*), as we know, does not obey the laws of refraction. In general, therefore, the angle of refraction of the *O*-ray will not be in same plane with the angle of refraction of the *E*-ray.

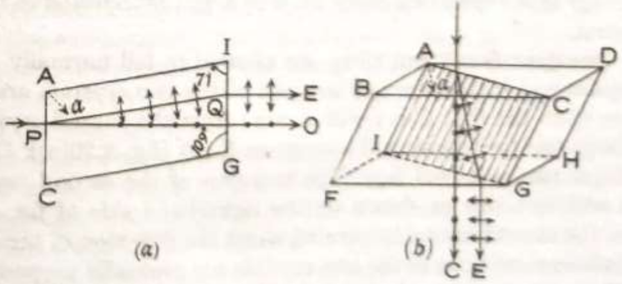

Fig. 4.19

If the emergent *O*-ray and *E*-ray are analysed with suitable analyser it will be found that both the rays are plane–polarised. The *O*-ray is polarised in the principal section of the crystal *i.e.*, its vibrations are perpendicular to the principal section. In fig. 4.19(a) and (b), these vibrations have been represented by dots on the ordinary ray. The *E*-ray, on the other hand, is polarised perpendicular to the principal section of the crystal *i.e.*, its vibrations are parallel to the principal section. The nature of vibrations of the E-ray has been shown in the fig. 4.19(a) and (b) by small straight lines with opposite arrow heads. In simple words, it may be said that *the vibrations of O-ray are perpendicular to the optic axis while those of E-ray are parallel to the optic axis of the crystal.*

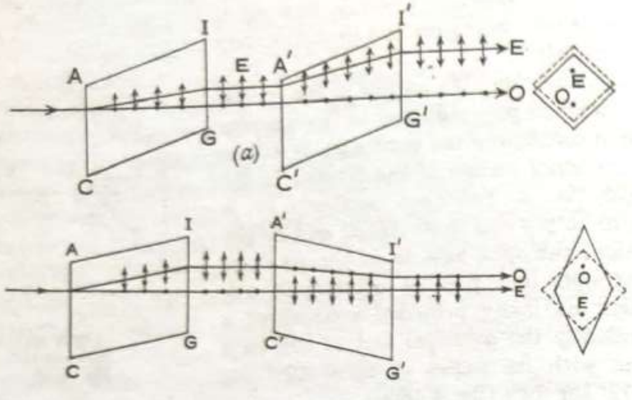

Fig. 4.20

If the emergent *O*-ray and E-ray are allowed to fall normally on a similar calcite crystal and if the principal sections of the two crystals are parallel to each other, then the first *O*-ray will emerge from the second crystal as *O*-ray and similarly the first *E*-ray will emerge as *E*-ray [fig. 4.20(a)]. Consequently we would get two polarised rays. The end-view of the second crystal, in this condition will look like as shown on the right–hand side of fig. 4.20(a).

If, now, the second crystal be rotated about the direction of incident light so that the principal sections of the two crystals are mutually perpendicular, then the first *O*-ray on entering the second crystal will become the *E*-ray and vice versa [fig. 4.20(b)]. If the principal sections of the two crystals make any other angle between 0° and 90°, we will get four rays, because both the first *O*-ray and *E*-ray will now suffer double refraction. The intensities of these four rays will depend upon the angle included between the principal sections. If the angle included be 45°, the intensities of all the four rays will be equal.

❑ 4.16. Nicol prism :

We have seen that a calcite crystal or a polaroid produces polarised light and that they can also be sued to detect such light. William Nicol designed a crystal of icelandspar which was widely used for producing and detecting polarised light. It is known as *Nicol prism*. Its basic principle is to divide a ray into

ordinary and extraordinary rays by double refraction, both being plane polarised and then to allow only the polarised *E*-ray to emerge, eliminating the *O*-ray by total internal reflection.

Construction : The prism is made from a piece of icelandspar crystal about three times as long as it is wide. *ABCD* is such a crystal [fig. 4.21(a)]. The corners *B* and *D* are the blunt corners of the crystal. In a natural crystal, the surfaces *AB* and *CD* make angles of about 71° with the arms *AD* and *BC* respectively. The faces are then cut and polished in such a way that the new polished surfaces *AI* and *CF* make angle of 68° with the arms *AF* and *CI* respectively. *AICF* represents the principal section of the crystal. The calcite crystal is then cut from one blunt corner to another into two halves along the diagonal *IF* and the halves are again cemented together by a layer of *canada balsam*. The principal section *AICF* of the crystal takes up an appearance as shown in fig. 4.21(b). It is clear that in this case, $\angle AIF = \angle CFI = 90°$. The reason for selecting canada balsam as the adhesive substance is that it has a refractive index (μ_b) of about 1.55 for both ordinary and the extraordinary rays while the refractive index of the crystal for the ordinary rays (μ_0) is 1.658 and 1.486 for the extraordinary rays (μ_e). So, $\mu_0 > \mu_b > \mu_e$. Thus, a critical angle exists between the crystal and the canada balsam for the ordinary rays but not for the extraordinary rays. The edges of the prism are painted black in order to absorb the reflected rays.

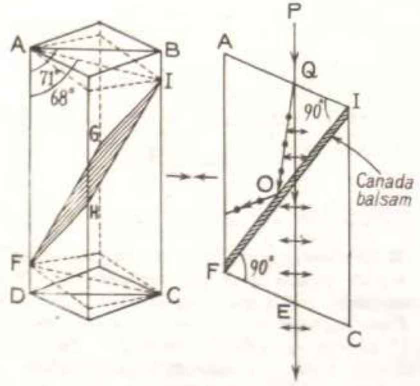

Fig. 4.21

Action : When an unpolarised beam *PQ* is incident on the surface *AI* of the crystal normally, it is split up into *O*-ray and *E*-ray in the crystal [Fig. 3.21(b)], both of them being plane–polarised. The vibrations of *E*-ray are parallel to the principal section *AICF* while those of *O*-ray perpendicular to it. As there exists a critical angle of 69°* between the crystal and the canada balsam for the ordinary ray which is incident on the surface of the canada balsam at an angle greater than the aforesaid critical angle due to the length of the crystal being about three times its width, the ordinary ray suffers total internal reflection and is directed towards the edge of the prism where it is absorbed by the black pigment. The emergent light in this case, is due to the extraordinary ray and is polarised. In this way, the Nicol prism acts as a polariser.

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* The refractive index of canada balsam for ordinary ray is $\mu = \frac{1.658}{1.550}$

$\therefore \sin \theta_c = \frac{1}{\mu} = \frac{1.550}{1.658}$, θ_c being the critical angle.

$= 0.935 = \sin 69°$ (nearly) $\therefore \theta_c = 69°$.

Nicol as an analyser : Of two identical Nicol prism, one may be used as a polariser and the other as an analyser of the polarised beam. In fig. 4.22, P is the polarising Nicol and A is the analyser.

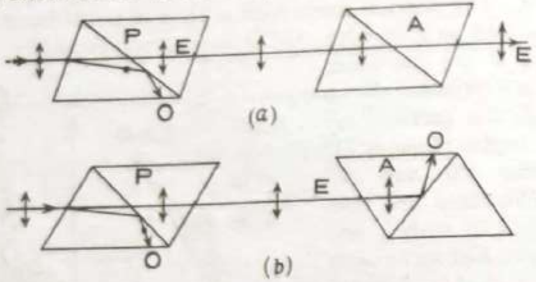

Fig. 4.22

[**Note** : The extraordinary ray has also a limit beyond which it is internally reflected at the canada balsam layer like the ordinarhy ray. μ for extraordinary ray is 1.486 when the extraordinary ray is travelling at right angles to the direction of optic axis of the crystal. If the E-ray happens to travel along the optic axis, its refractive index becomes equal to that of the O-ray which is 1.658. Depending on the direction of propagation of E-ray, μ_e lies between 1.486 and 1.658. Therefore, in a particular case μ_e may be greater than 1.550 and the angle of incidence will be greater than the critical angle. Then the E-ray will also be totally reflected at the canada balsam layer and will be absorbed by the black paint with which the outside surfaces of the crystal are coated.]

When E-ray comes out of the polarising Nicol, it becomes plane–polarised with vibrations confined to the principal section of the Nicol. When the principal section of the analysing Nicol A is parallel to that of the polarising Nicol, the ray enters into the second Nicol as E-ray [fig. 4.22(a)] and is transmitted out. But if the analyser be rotated about the line of propagation of light, so that the two principal sections are perpendicular to each other, then the E-ray coming out of the polariser will enter the analyser as O-ray which will be totally reflected by the canada balsam layer towards the edge of the prism and will be absorbed by the balck paint. Hence, no light will emerge from the analyser. The analyser, in this positions, is crossed with the polariser. In this way, a Nicol can act as a detector of polarised beam.

❐ 4.17. Polarisation by scattering :

When a beam of light is scattered by very small particles, the scattered light is found to be partially polarised. The degree of polarisation is greatest for light scattered in a direction at right angles to the main beam, but this is usually far from complete. Light from the blue sky, being the result of scattering by small atmospheric particles, is partially polarised.

❐ *4.18. Huygen's theory of double refraction :

Based on some experimental findings, Huygens developed a theory of double reflection of light. The findings are as follows :

(i) A point inside a doubly refracting crystal acts as a source of two secondary wavelets—one corresponding to the ordinary ray and the other to the extraordinary ray.

(ii) As the O-ray obeys the laws of refraction,

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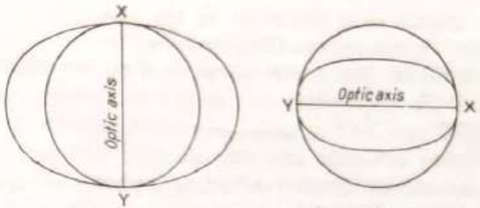
* For Burdwan University only.

$$\mu_0 = \frac{\sin i}{\sin r_0} = \frac{V}{V_0} = \text{constant.}$$

It is easy to understand that the velocity of O-ray is same in all directions in the crystal and hence the wave–surface due to O-ray will be spherical in shape *i.e.*, the surface will be a spheroid. On the other hand, the E-ray does not obey the laws of refraction ; hence for the E-ray, $\frac{\sin i}{\sin r_e} = \frac{V}{V_e}$ is not a constant. It takes up values according to the direction of propagation of the ray. This shows that the velocity V_e of the E-ray is different in different directions. Considering this fact, Huygens supposed that the wave–surface due to E-ray is an ellipsoid.

(iii) As the velocities of O-ray and E-ray are equal along the optic axis and as these rays travel along the same direction when incident parallel to the optic axis, it is clear that the two wave–surfaces should touch each other at points intersected by the optic axis. [The points X and Y in fig. 4.23(a) and (b)].



Nigative crystal (a)

Positive crystal (b)

Fig. 4.23

(iv) When the ray is incident normal to the optic axis, the O-ray and the E-ray travel along the same direction but with different velocities. The difference in velocities, in this case, becomes the greatest.

From the above experimental facts Huygens came to the conclusion that the wave–surface of the secondary wavelets inside uniaxial crystals may be ellipsoidal or spheriodal. Ellipsoidal wave–surface is caused due to E-ray while spheriodal wave–surface is caused due to O-ray. The ellipsoid and the spheriod touch each other at two points and the line joining the points gives the direction of the optic axis. In the case of negative crystals where the velocity of the O-ray is less than that of E-ray, the spheriod is enclosed within the ellipsoid and the minor axis XY of the ellipse coincides with the optic axis of the crystal [Fig. 4.23(a)]. In the case of positive crystal where the velocity of O-ray is greater than that of E-ray, the spheroid encloses the ellipsoid and the major axis XY of the ellipse coincides with the optic axis of the crystal [Fig. 4.23(b)]. We shall now discuss how Huygens explained the phenomenon of double refraction with his theory, remembering the fundamental postulate that every point on a wavefront acts as a source of secondary wavelets. Some specific cases of uniaxial negative crystal will be considered as illustrations of Huygen's theory :

(a) *Double refraction through a calcite crystal cut in such a way that the optic axis lies in the plane of incidence but not parallel to the plane of refraction.*

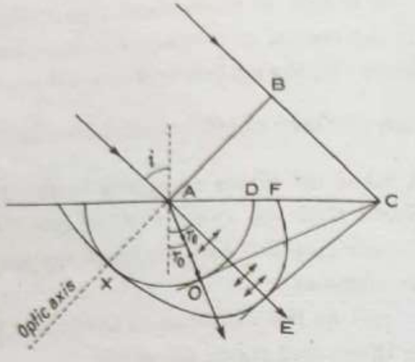


Fig. 4.24

Consider a plane wavefront *AB*, perpendicular to the plane of paper incident obliquely on the surface *AC* of a doubly refracting crystal [Fig. 4.24]. The surface *AC* is perpendicular to the plane of the paper. *AX* is the optic axis of the crystal which, in the present case, lies in the plane of incidence *i.e.,* in the plane of the paper. As the wavefront touches the point *A* on the crystal, two secondary wavelets due to double refraction originate from the point. By the time, the wavelets from the point *B* of the wavefront just reach the point *C* on the crystal, the spheroidal wave–surface and the ellipsoidal wave–surface originating from *A*, will arrive at the positions *XOD* and *XEF* respectively, touching each other at the point *X*. The sections of the spheroid and the ellipsoid will evidently be semi–circle and ellipse respectively. Draw two tangents planes *CO* and *CE* from *C* to the two wave–surfaces. The plane passing through the points *C* and *O* perpendicular to the plane of paper represents the ordinary refracted wavefront and *AO* represents the ordinary ray. The ordinary ray *AO* lies in the plane of the paper and the ratio of sine of the angle of incidence (*i*) to the sine of the angle of refraction (r_0) is always a constant. The ratio evidently does not depend on the direction of the optic axis.

The tangent plane *CE* drawn from *C* to the ellipsoidal wavefront *XEF* represents another refracted wavefront, the corresponding refracted ray being *AE*. This is the extraordinary ray. It may or may not remain in the plane of incidence (*i.e.,* the plane of the paper) and may not always make same angle (r_e) with the normal to the refracting surface at *A* (shown by dotted vertical line). This depends on the direction of the optic axis.

Since the refractive angle of a ray in a medium depends on the velocity of light in the medium, it follows that the velocity of *E*-ray in the crystal depends on the direction of propagation relative to the position of the optic axis. When the *O*-ray and the *E*-ray both travel along the optic axis *AX*, their velocities are equal. In that case, the ray is not polarised and emerges as a single ray. In other words, *double refraction does not take place if the ray travels along the optic axis.* When the ray is incident normal to the optic axis, they (*i.e., O*-ray and *E*-ray) travel along the same direction but with different velocities. They are also polarised in mutually perpendicular planes.

(b) *When the optic axis lies in the plane of incidence and parallel to the refracting plane :*

Since the optic axis is parallel to the surface of the crystal and lies in the

plane of incidence, it is represented by the line *XY* [Fig. 4.25]. Following the method described earlier, the tangent planes *CO* and *CE* represent the ordinary and the extraordinary refracted wavefronts. *AO* represents the corresponding refracted *O*-ray and *AE* the refracted *E*-ray. They travel in different directions and with different velocities. Here, the ellipsoid and the spheroid touch each other at *X* and *Y* because the line *XY* represents the optic axis. It is evident from fig. 4.25 that *E*-wavefront is always ahead of the *O*-wavefront.

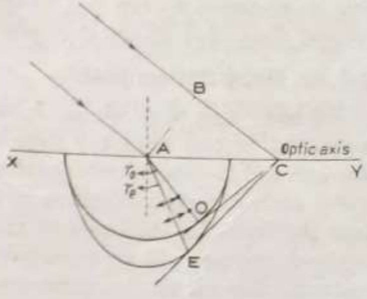


Fig. 4.25

(c) *When the optic axis lies in the refracting plane perpendicular to the plane of incidence :*

Since the optic axis is perpendicular to the plane of incidence (*i.e.,* the plane of the paper), the sections of the *O*-wavefront and *E*-wavefront in the plane of the paper will be circular [Fig. 4.26]. Tangent planes drawn from *C* to the wave–surfaces represent the refracted wavefronts. Thus, *CO* represents the *O*-wavefront and *CE* the *E*-wavefront. Fig. 4.26 shows that *E*-ray *AE* and the *O*-ray *AO* travel along different directions with different velocities. The figure also shows that the section of the *E*-wavefront in the plane of incidence is circular indicating that E-ray travels in all direction with equal velocity. Hence, *E-ray, in this case, obeys the laws of refraction.* It is to be noted that here also, the *E*-wavefront is ahead of the *O*-wavefront.

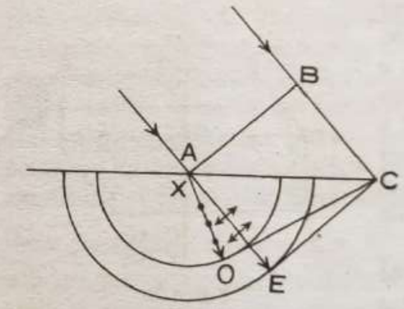


Fig. 4.26

(d) *When the ray is incident normally on a crystal whose optic axis is parallel to the crystal surface :*

A plane wavefront *AB* is incident on the crystal surface whose optic axis is *XY*. The incidence is evidently normal. In this case, the *O*-wavefront and the *E*-wavefront will be parallel to each other. In fig. 4.27, *OO′* is the refracted *O*-wavefront and *EE′* the refracted *E*-wavefront. As a result, the ordinary and the extraordinary rays also travel parallel to each other. Although the rays will not be separated from each other, yet double refraction will take place due to difference in their velocities. The *E*-ray will come out of the crystal first and then the *O*-ray, creating a path difference and hence a phase-difference between the two rays.

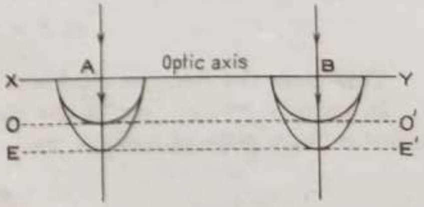


Fig. 4.27

In the same way, wave-surface may be drawn according to Huygen's theory for a positive crystal. Only difference is that for a positive crystal, the ellipsoid will be enclosed within the spheroid because the velocity of ordinary ray is greater than that of the extraordinary ray in a positive crystal.

❒ 4.19. Retardation plates :

We have seen that, we can have two plane–polarised beams with vibrations contained in two mutually perpendicular planes if a ray is allowed to pass through a thin slice of doubly refracting crystal. Since the *O*-ray and the *E*-ray travel with different velocities in the crystal, they will take different times to pass through the thickness of the crystal and hence a path–difference or a phase–difference will be introduced between them. The phase–difference will evidently depend on the thickness of the plate. For this reason thin plates of doubly refracting crystals are called *retardation plates*. The two common types of retardation plates that are frequently used are : (i) *quarter–wave plate* and (ii) *half–wave plate.*

(i) **Quarter-wave plate :** A crystal plate which is cut so that it produces a path-difference of $\lambda/4$ (*i.e.*, quarter-wave) or a phase–difference of $\pi/2$ between the *O*-ray and the *E*-ray is called a *quarter-wave plate.*

Consider a crystal which is cut in such a way that the optic axis lies on the surface *ABCD* and is parallel to it [Fig. 4.28]. Let a plane-polarised light-ray *PQ* be incident normally on the crystal. Suppose, the plane of vibration of the polarised beam makes an angle θ with the optic axis of the crystal.

It is evident that as the beam enters the crystal, it is divided into *O*-ray & *E*-ray. Since the incident ray is normal to the optic axis, both the rays travel along the same straight line but with different velocities. In negative crystals like calcite etc., $\mu_0 > \mu_e$. If t be the thickness of the plate, the optical paths traversed by each ray in passing through the thickness of the plate are $\mu_0 t$ and $\mu_e t$ respectively. Hence, the path-difference introduced between the *O*-ray and *E*-rays after emergence $= t(\mu_0 - \mu_e)$. For a quarter-wave plate $t(\mu_0 - \mu_e) = \lambda/4$.

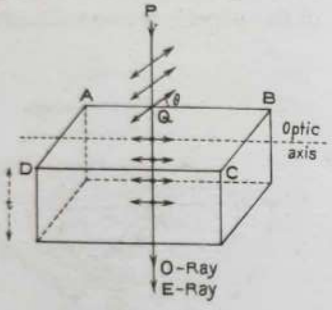


Fig. 4.28

$$\text{or, } t = \frac{\lambda}{4(\mu_0 - \mu_e)}$$

This gives the required thickness of a quarter-wave plate.

In positive crystals like quartz etc., $\mu_e > \mu_0$; so if t be the thickness of a quarter-wave plate of positive crystal, then, $t(\mu_e - \mu_0) = \lambda/4$

$$\text{or, } t = \frac{\lambda}{4(\mu_e - \mu_0)}$$

(ii) **Half-wave plate :** A crystal plate cut so that it produces a path-difference of $\lambda/2$ (*i.e.*, half-wave) or a phase-difference of π between the *O*-ray and the *E*-ray, is called a *half-wave* plate.

As before, for a half-wave plate of negative crystal, $t = \dfrac{\lambda}{2(\mu_0 - \mu_e)}$ and for a half-wave plate of positive crystal, $t = \dfrac{\lambda}{2(\mu_e - \mu_0)}$.

Usually very thin mica or quartz plate is used to prepare a quarter-wave or a half-wave plate and its thickness (t) is measured with the help of above formula using sodium light ($\lambda = 5896 \times 10^{-8}$ cm).

Uses : Quarter-wave and half-wave plates are used to produce and to analyse circularly polarised and elliptically polarised beams of light.

❒ 4.20. Circularly polarised light :

We have already seen that if in a *polarised* beam of light, the vibrating particles can be thought of as constrained to described a circular path, the corresponding light is referred to as *circularly polarised light.* The amplitude of vibrations in a circularly polarised light remains constant but the directions change.

If a plane–polarised light is shone on to a quarter-wave plate so that the direction of vibration is at 45° to the optic axis of the crystal plate, the components are equal and circularly polarised light results.

We know that two *rectangular linear* vibrations of same amplitude but of phase–difference $\pi/2$, when superposed, give rise to a circular vibration. Hence, a circularly polarised beam may be supposed to be a combination of two such plane–polarised beams. Now, the quarter-wave plate will refract the incident beam doubly into *O*-ray and *E*-ray whose vibrations are confined in two mutually perpendicular planes. Further, since the vibrations of the incident polarised beam make an angle of 45° with the optic axis of the plate, the amplitude of *O*-ray and E-ray will be equal. On emergence from the quarter-wave plate, their phase-diff. $= \frac{2\pi}{\lambda} \times$ path-diff. $= \frac{2\pi}{\lambda} \times \lambda/4 = \frac{\pi}{2}$. Hence, they will produce a circularly polarised light on emergence. If circularly polarised beam is viewed through a Nicol prism which is slowly rotated, no variation in transmitted intensity can be seen. The reason is that when the Nicol is rotated, the intensity of one plane-polarised component decreases as much as the intensity of the other component increases. Under these circumstances, circularly polarised light cannot be distinguished from ordinary light.

Distincition between circularly polarised and unpolarised beams :

To distinguish between circularly polarised light and ordinary unpolarised light, a quarter-wave plate should be placed between the source of light and the analysing Nicol. If the beam is circularly polarised then on entering the

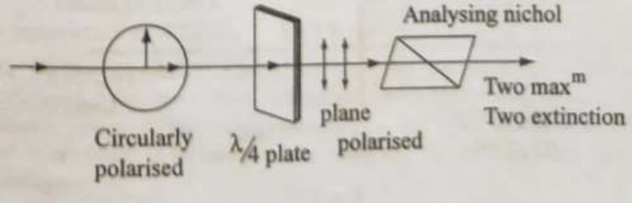


Fig. 4.29

$\lambda/4$ plate, it will be split up into two rectangular components of equal amplitude – one parallel and the other perpendicular to the axis of $\lambda/4$ plate and thus differing in phase by $\pi/2$. A further phase–difference of $\pi/2$ is introduced between the components on passing through the $\lambda/4$ plate. Thus, the total phase–difference between the two rectangular components on emergence will be $\pi/2 \pm \pi/2 = \pi$ or 0. In either case the emergent light is plane–polarised (Fig. 4.29).

Now, if the emergent light be examined by a rotating Nicol, *its intensity will change in intensity from maximum to zero twice during each rotation.* On the other hand, if the incident beam be ordinary (unpolarised) light, then after passing through the quarter–wave plate, the beam will remain unpolarised and will not be extinguished by the analyser. The analysing Nicol will simply convert it into a plane–polarised beam and will not produce any variation in intensity when rotated.

☐ 4.21. Elliptically polarised light :

We know that if, in a *polarised* beam of light, the vibrating particles can be thought of as constrained to describe an elliptical path, the corresponding light is referred to as *elliptically polarised light.* In elliptically polarised beam, both the amplitude and direction of vibration continually change.

If a plane–polarised light is shown on to a quarter–wave plate so that the direction of vibration is inclined at any angle other than 45° to the optic axis of the plate, elliptically polarised light will result with the major axis or minor axis of the characteristic ellipse parallel to the optic axis of the plate :

We know that two *rectangular linear* vibrations of unequal amplitude when superposed, in general, give rise to an elliptical vibration. Hence, an elliptically polarised beam may be supposed to be a combination of two such plane–polarised beams. If elliptically polarised beam is viewed through a Nicol prism which is slowly rotated, some variation of transmitted intensity can be seen but at no orientation of the analysing Nicol will there be complete extinction. It appears that a mixture of plane–polarised light and unpolarised light (or, a partially plane–polarised light) is incident on the analysing Nicol. Under these circumstances elliptically polarised light cannot be distinguished from a partially plane–polarised light.

Distinction between elliptically polarised and partially plane–polarised beams :

To distinguish between the above two types of beam, the light should be allowed to pass through a quarter–wave plate before it falls on the analysing Nicol. The quarter–wave plate should be so placed that its optic axis is directed along the major or minor axis of the characteristic ellipse. The elliptically polarised beam on entering the $\lambda/4$ plate, is split up into two unequal components vibrating perpendicular to each other and thus having a phase–difference of $\pi/2$. The $\lambda/4$ plate introduces an additional phase–difference of $\pi/2$ between the two components so that the total phase–difference now becomes $\pi/2 \pm \pi/2 = \pi$ or 0. So, *the elliptically polarised beam becomes plane– polarised after passing through $\lambda/4$ plate.* The emergent light when examined through a rotating Nicol will give two maxima and two complete extinction during each rotation. On the other hand, if incident beam is partially plane–polarised, no change in its nature will be produced by the quarter–wave plate and hence in no orientation of the analysing Nicol, will the beam be extinguished.

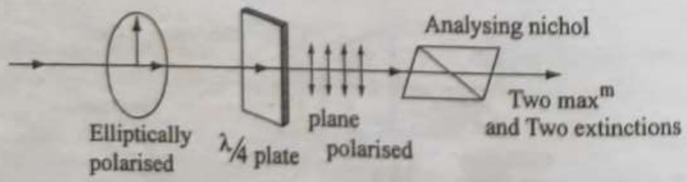


Fig. 4.30

☐ 4.22. Mathematical treatment of production of circularly polarised and elliptically polarised beams :

Consider a plane polarised light, incident normally on a plate of uniaxial crystal cut parallel to the optic axis. The component B, with its vibrations parallel to the optical axis (Fig. 4.31) will travel through the crystal as E-ray and the component A as O-ray. It follows that when the components leave the crystal plate, there will be a phase–difference between them as the velocities of the two components are not equal. The optical path difference between the components after leaving the crystal = $t(\mu_0 - \mu_e)$ where t is the thickness of the plate.

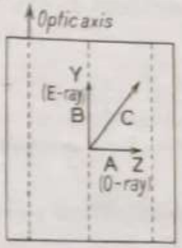


Fig. 4.31

$$\therefore \text{Phase–difference } \delta = \frac{2\pi}{\lambda} \times \text{path–diff.} = \frac{2\pi}{\lambda} t\,(\mu_0 - \mu_e). \text{ where}$$

λ is the wavelength of light in free space.

Thus, we have to consider the summation of two S.H.M.'s whose vibrations are at 90° to each other with a phase–difference δ. If we consider the light emerging from the crystal plate to be travelling in the x-direction, then the transverse vibration components of which it is composed may be written as :

$$y = B \cos(\omega t - k.x) \text{ or, } y = B \cos C \quad \text{... ... (i)}$$

$$\text{and } z = A \cos(\omega t - kx + \delta) \text{ or, } z = A \cos(C + \delta) \quad \text{... ... (ii)}$$

From (i) $\cos C = y/B$.

From (ii) $z/A = \cos C, \cos\delta - \sin C \sin\delta$

$$\therefore y/B \cos\delta + \sqrt{1 - y^2/B^2}\,\sin\delta = z/A.$$

$$\text{Squaring, } (1 - y^2/B^2)\sin^2\delta = \frac{z^2}{A^2} - \frac{2yz}{A.B}\cos\delta + \frac{y^2}{B^2}\cos^2\delta$$

$$\text{or, } (1 - y^2/B^2)\sin^2\delta - (y^2/B^2)\cos^2\delta = z^2/A^2 - (2yz/A.B)\cos\delta$$

$$\text{or, } \sin^2\delta = \frac{z^2}{A^2} - \frac{2yz}{A.B}\cos\delta + \frac{y^2}{B^2}.$$

This is the equation of an ellipse.

If $\delta = \pm\,^\pi/_2$, we have, $1 = z^2/A^2 + y^2/B^2$ and therefore the major and minor axes of the ellipse lie along y and z directions. Now light of the above type emerging from the plate is said to be *elliptically polarised.* For the special case of $\delta = \delta \pm \pi/2$ and $A = B$, the equation becomes $y^2 + z^2 = A^2$ which is an equation of a circle and the corresponding light is said to be *circularly polarised.*

☐ 4.23. General discussion on different types of polarised light :

What can be the different nature of light in respect of polarisation ? Light can be (i) unpolarised (ii) plane–polarised (iii) circularly polarised (iv) elliptically polarised or (v) partially plane–polarised. How can we find out the real nature ? We can find the real nature by allowing the light to fall on a Nicol and noticing the variation of intensity of transmitted beam when the Nicol is slowly rotated about the line of propagation of light. If the Nicol can extinguish the light completely at a particular orientation, the original beam is plane–polarised. If there is no intensity variation of the transmitted beam as the Nicol is rotated, the original beam is either circularly polarised or unpolarised. If, on the other hand, some intensity variation of the transmitted beam is found but in no position the light is completely cut off by Nicol, the original beam is either elliptically polarised or partially plane–polarised. To memorise the results may be tabulated as follows :

Nature of light incident on the Nicol	**Variation of intensity of the transmitted light as the Nicol is slowly rotated.**	**Inference**
(i) Unpolarised or circularly polarised	(i) No variation of intensity	(i) Original light is unpolarised or circularly polarised.
(ii) Plane–polarised	(ii) Variation of intensity and complete extinction at a given position	(ii) Original light is plane–polarised.
(iii) Partially plane–polarised or elliptically polarised.	(iii) Variation of intensity but no extinction at any position of the rotating Nicol.	(iii) Original light is partially plane–polarised or elliptically polarised.

The alternative nature that has been mentioned in the inference of (i) and (iii) may be finally settled with the help of a quarter–wave plate as mentioned arts. 4.21 and 4.22. So a beam of light, whatever may be its nature or state of polarisation, can be fully analysed with the help of a Nicol and a quarter–wave plate.

□ 4.24. Babinet's compensator :

One of the drawbacks of a quarter–wave plate or a half–wave plate is that it produces only a fixed path–difference between the *E*-ray and the *O*-ray and can be used only for light of given wavelength. For different wavelengths different quarter–wave plates or half-wave plates are to be used. This inconvenience has been removed by designing a compensator by means of which a desired path–difference can be introduced between the *E*-ray and the *O*-ray. The compensator is known as *Babinet's compensator* after the name of its inventor Babinet.

Fig. 4.32 shows the section of the compensator. It consists of two small wedge–shaped pieces of quartz A and B having small angles. The optic axis of the quartz A is parallel to the crystal surface while that of the quartz B is perpendicualr to the surface. The plates have, therefore, their optic axes mutually perpendicular. As a result, the *E*-ray in A will behave as ordinary ray in B-while the *O*-ray in A will behave as extraordinary ray in B. Suppose, a plane polarised light is incident normally at P on the prism A. The ray will be broken into *E*-ray and *O*-ray in the crystal and they will travel along the same direction with different velocities. The path–difference introduced between the rays after they have travelled a distance $PQ = (t_1)$ in the prism A is given by

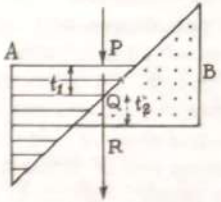


Fig. 4.32

$$\delta_1 = (\mu_e - \mu_0)t_1.$$

The rays then enter into the prism B where *E*-ray becomes the ordinary ray and *O*-ray becomes the extraordinary ray because the optic axes of the prisms are mutually perpendicular. When the rays travel a distance QR $(= t_2)$ in the crystal B, the path–difference introduced in given by $\delta_2 = (\mu_0 - \mu_e)t_2$.

∴ Total path–difference between the rays

$$\begin{aligned} \delta &= \delta_1 + \delta_2 \\ &= (\mu_e - \mu_0)t_1 + (\mu_0 - \mu_e)t_2 \\ &= (\mu_e - \mu_0)\ (t_1 - t_2) \end{aligned}$$

Hence, the phase–difference between the rays $= \frac{2\pi}{\lambda} \times \delta$

$$= \frac{2\pi}{\lambda}(\mu_e - \mu_0)\ (t_1 - t_2).$$

The crystals A and B are so mounted that A is fixed and B can move along the side of A with the help of a rack and pinion arrangement. Sliding the crystal B, $(t_1 - t_2)$ can be given any desired value and hence any desired path–difference or phase–difference may be introduced between the rays.

If, however, $t_1 = t_2$ *i.e.*, $PQ = QR$, the net phase difference is zero and the incident plane–polarised beam emerges from the compensator without any change in its nature. On either side of the direction PQR, the phase–difference will evidently vary symmetrically.

□ 4.25. Rotation of plane of polarisation ; Optical activity :

We know that if two nicols N_1 and N_2 be kept crossed and a beam of monochromatic light be sent through them, no light emerges from the second nicol N_2. Light, in passing through the nicol N_1 is plane–polarised with vibrations parallel to the principal section of N_1 but as the principal section of N_2 is perpendicular to these vibrations, the nicol N_2 obstructs the light [Fig. 4.33(a)]. If now, a calcite plate

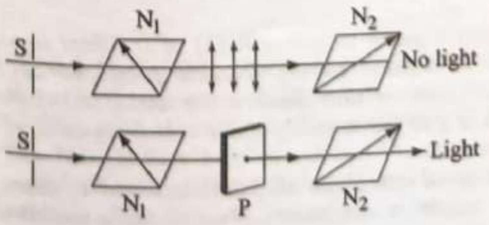


Fig. 4.33

cut with its optic axis perpendicular to its refracting face, be interposed between N_1 and N_2, no change takes palce *i.e.*, light remains obstructed by N_2 as before. This shows that the calcite plate cannot bring about any change in the state of polarisation of a beam. If instead of the calcite plate, a similarly cut quartz plate P be interposed, some light will come out of the nicol N_2 [Fig. 4.33(b)]. The light can be completely extinguished by turning the nicol N_2 a little more from the crossed position. This shows that the beam after coming out of the quartz plate remains plane–polarised but the plane of polarisation has to some extent turned through. The angle through which the analysing nicol N_2 has to be rotated further in order to obtain complete extinction of light, gives the angle of rotation of the plane of polarisation of the beam.

This property of rotating the plane of polarisation of a plane–polarised beam is known as *optical activity*. The substances which bring about optical activity are called *optically actives* substances. Quartz is an optically active substance but quartz when fused loses its optical activity. This shows that *optical activity depends upon the molecular arrangement of the substance.*

Optically active substances are divided into two classes :

(i) *Dextro–rotatory* : Rotation is said to be dextro or right–handed if one looking along the path of the beam towards its source, the rotation appears to be clockwise. The substances causing dextro–rotation are known as dextro–rotatory substances. Some quartz crystals, cane sugar etc., are the examples.

(ii) *Laevo–rotatory* : Rotation is said to be leavo or left–handed if on looking along the path of the beam towards the source, the rotation appears to be anti–clockwise. The substances causing laevo-rotation are known as laevo–rotatory substances. Some form of quartz crystals, fruit–sugar etc., are the examples.

Besides crystals, liquids like turpentine, sugar solution etc., also exhibit optical activity. There are some organic substances whose chemical properties are identical but optical activity is opposite. Quartz crystals are available in two forms whose optical activity is opposite but otherwise they are exactly alike.

☐ 4.26. Specific rotation :

Rotation of plane of polarisation produced by an optically active substance depends on :

(i) the length of the substance traversed by the beam.
(ii) the density (if solid) or concentration (if solution) of the susbtance.
(iii) wavelength of the incident light.
(iv) the temperature of the substance.

Optical activity of a substance is measured by *specific rotation* which is defined as follows :

For a given temperature (t) and a given wavelength (λ) of incident light, specific rotation of a substance is defined as the rotation produced by 1 decimeter (i.e., 10 cm) of the substance of unit density (for solid) or of the solution of unit concentration (i.e., 1 g of the optically active substance in 1 cm³ of the solution).

Suppose, θ is the total rotation of the plane of polarisation of a plane–polarised beam when it travels a length of l decimeter of a substance (solid) of density ρ. In this case,

the specific rotation, $[S]_t^{\lambda} = \dfrac{\theta}{l.\rho}$

Similarly, if c be the concentration of a solution (*i.e.*, 1 g of the substance present per cm^3 of the solution), the solvent being assumed inactive,

$$[S]_t^{\lambda} = \frac{\theta}{l.c}.$$

☐ 4.27. Rotatory dispersion :

It has been mentioned in the previous article that the amount of rotation of the plane of polarisation by an optically active substance depends on the wavelength of the incident light. As a matter of fact the rotation is approximately inversely proportional to the square of the wavelength *i.e.*,

$$\theta \propto \frac{1}{\lambda^2}$$

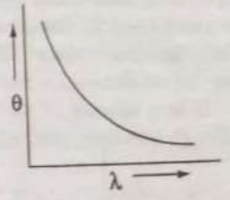


Fig. 4.34

Thus, the rotation is least for red (λ = 7000A°) and greatest for violet (λ = 4000 A°). If a graph is plotted between the wavelengths (λ) and corresponding rotations (θ), a curve of the form shown in fig. 4.34 is obtained. Thus different colours are rotated through different angles. Hence, if *white plane–polarised light* is passed through quartz along its optic axis, the field of view appears coloured *i.e.*, white light is decomposed into its constituent colours due to different optical rotation. This phenomenon is known as *rotatory dispersion.*

☐ 4.28. Polarimeters :

These are instruments for measuring the optical activity of liquid solutions. Measurement of optical activity has been found useful and wide applications in determining the percentage of sugar in a given solution, in the urine of a diabetic patient etc. In sugar factory, the instrument is used to ascertain the percentage of sugar present in cane juice. The instrument is known as *Saccharimeter* when they are solely intended for sugar analysis.

Fig. 4.35 shows the simplest type of a polarimeter. It consists of two nicols N_1 and N_2 capable of rotating about a common axis. A tube T, with mouths closed by plane glass plates is interposed between the nicols. The tube T may be filled with the optically active solution. S is a source of monochromatic light. The tube is first kept empty and the analyser N_2 is rotated slowly to get the complete extinction of the emergent beam. The tube is then filled with the solution. The field of view of the telescope E now brightens up a little. The analysing nicol N_2 is again rotated slowly till the field of view is darkened. The

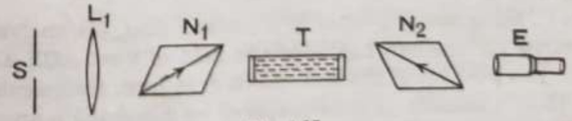


Fig. 4.35

difference between the two positions of the analyser gives the angle of rotation of the plane of polarisation.

As it stands, however, the instrument as described above is not very accurate due to difficulty in setting the analyser to the exact extinction point. Many devices have been introduced to increase sensitivity of setting the analyser, the most important being *half-shade plate* devised by Laurent. The half-shade plate is used with a given wavelength of light, usually the sodium light.

Half-shade plate : It is a round plate (fig. 4.36), one half of which is made of quartz (*AQB*) and the other half glass (*APB*). The quartz plate is cut with its optic axis parallel to the line *AB* and its thickness is such that it introduces a path-difference of λ/2 between the *O*-ray and *E*-rays. The quartz plate in fact, is a half-wave plate. The thickness of the glass is such that it absorbs the same amount of light as the quartz plate does. Fig 4.37 shows the arrangement of *Laurent's half-shade polarimeter.*

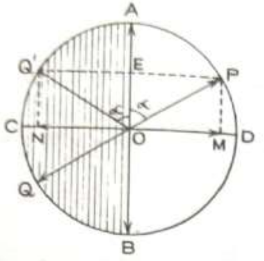


Fig. 4.36

S is a source of monochromatic light suitable for the half-shade used. The

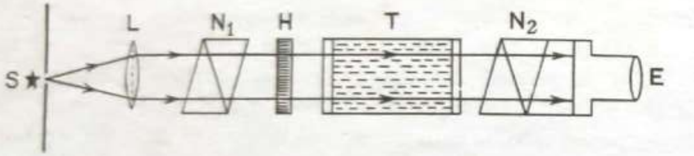


Fig. 4.37

rays of light are rendered parallel by the convex lens *L* and they are incident normally on the half-shade plate *H* after passing through the polarising nicol N_1. *T* is a tube with mouths closed by flat glass plates. It may be filled by optically active liquid. The light after travelling the length of the solution falls on the analysing nicol N_2, behind which is placed a telescope *E*. Light emerging from the analyser will be visible through the telescope. The analysing nicol N_2 and the telescope *E* are enclosed in a tube which can rotate about the axis of the whole instrument. The angle of rotation is measured with the help of a circular scale and a vernier.

Procedure : Taking distilled water in the tube *T* first, the analysing nicol N_2 is slowly rotated till the two halves of the field of view *AQB* and *APB* are equally dark. The position of the analyser is read from the circular scale and the vernier. Then a solution is to be prepared by dissolving a known quantity of the substance (say, sugar) in distilled water, and the tube *T* is filled up with the solution. Looking through the telescope it will be seen that the two halves of the field of view are no longer equally dark. One half will appear brighter than the other. The analyser N_2 is rotated further till the two halves of the field of view are again equally dark. The difference of the two positions of the analyser gives the angle of rotation.

Principle of half-shade plate :

In order to understand the working of the half-shade plate, let us consider a plane-polarised beam of light incident normally on the plate in such a way that the plane of vibration is parallel to *OP* (Fig. 4.35). It is evident that the component of vibration incident on the *glass plate i.e., OP* will pass through it as such. The other component incident on the half-wave plate *AQB* will be broken up into *O*-ray and *E*-ray with vibrations parallel to *OC* and *OB* respectively. As these components pass through the half-wave plate, a phase-difference of π is introduced between them. As a result, when they come out we can consider that the vibrations are parallel to *OA* and *OC*. They will give rise to a plane-polarised beam with vibrations parallel *OQ'*. Both the vibrations *OQ'* and *OP* make equal angles with *AB*. Instead of the incident rays with vibrations parallel to *POQ*, we have now emergent ray with vibrations along *OP* and *OQ'*, *OQ'* and *OP* make equal angles with *AB*, the optic axis. If the principal section of the analyser nicol is set parallel to *AB*, the components of *OP* and *OQ'* parallel to *AB* (*i.e., PM* and *Q'N*) will have small but equal amplitude. As a result, the two halves of the field of view will be equally but feebly illuminated. This is referred to as two halves being *equally dark.*

If the principal section is, however, set perpendicular to *AB*, the components of *OP* and *OQ'* parallel to *CD* (*i.e., OM* and *ON*) will have equal but large amplitude. Consequently the two halves of the field of view will be *equally bright* [Fig. 4.36].

If the principal section of the analyser be set in any other position, one half of the field of view will be brighter than the other. For example, in fig. 4.38(i) and fig. 4.38(ii) the components of amplitude *ON* and *OM* are not equal. In the first position, *OM* is greater than *ON* while in the second position *ON* is greater than *OM*. This will produce unequal illumination of the two halves of the field of view. It goes without saying that during one complete rotation of the analysing nicol, the intensity will be twice maximum and twice minimum.

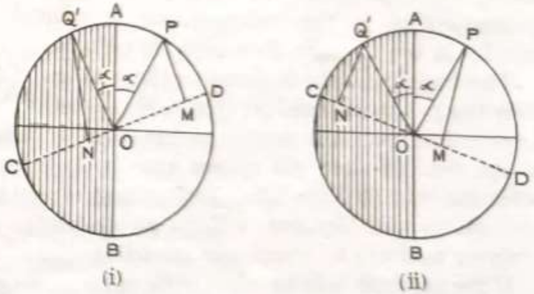


Fig. 4.38

Now, suppose that when the tube *T* containing distilled water is placed in its position, the two halves *AQB* and *APB* of the field of view are equally dark. In this condition, it is clear that the principal section of the analysing nicol N_2 is parallel to the line *AB* midway between *OP* and *OQ*. If the water in the tube *T* is now replaced by the solution under test, the plane of polarisation of the light will turn and some light will emerge from the nicol N_2. Consequently *OP* and *OQ* will turn to *OP'* and *OQ'* respectively (Fig. 4.39). This means that *AB* is no longer in the midway between *OP'* and

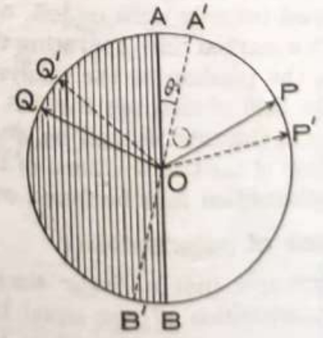


Fig. 4.39

OQ' and the two halves of the field of view will not longer be equally dark. To make the two halves equally dark again, the principal section of the analysing nicol N_2 should be turned to the position OA' so that it occupies a midway position between OQ' and OP'. It is evident that in this case, $\angle AOA' = \theta$ gives the angle of rotation of the plane of polarisation.

☐ 4.29. Biquartz polarimeter :

One of the disadvantages of half-shade plate is that it can not be used with any other wavelength of light except that for which it is prepared. Biquartz is free from this disadvantage as it can be used even with white light. Quartz crystal, we know, may be dextro-rotatory as well as laevo-rotatory. A biquartz consists of two semi-circular plates of quartz cut from right-handed and left-handed samples. The semi-circular portion APB in the fig. 4.40 is made of right-handed quartz and the other portion AQB by left-handed quartz. They are cut with their optic axes perpendicular to their refracting surfaces. The two are joined and cemented to from a circular plate. The thickness of each plate is nearly 3.75 mm, so that wavelength corresponding to the yellow colour is rotated equally in the opposite directions through 90°.

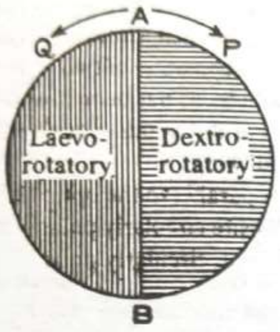


Fig. 4.40

The biquartz plate is placed in the position of the half-shade plate H, just after the polarising nicol N_1 (Fig. 4.36). If the analysing nicol N_2 be so set that its principal section is parallel to that of the polarising nicol N_1, the two halves of the field of view will appear equally dark because the vibrations of the polarised light (sodium light), having been turned 90° by the two semi-circular portions of the biquartz, will be perpendicular to the principal section of analyser and will be completely quenched.

If the incident light be *white*, different wavelengths of the incident light will have different rotation. Only the wavelength corresponding to the yellow light will have 90° rotation and will be totally obstructed by the analyser. The emergent light in which yellow light will be missing, will produce a dim grey-violet tint in both the halves, which is called *sensitive tint* or *tint of passage*.

If the position of the analyser is slightly turned towards right or left, one half will appear bluish and the other reddish with a marked line separating the two. In order to measure the specific rotation, the position of the analyser should be so adjusted that both the havles of the field of view get the tint of passage when the optically active liquid is taken in the tube as well as when the tube contains distilled water. From the readings of the two positions of the analyser, the angle of rotation of the plane of polarisation may be found out.

☐ *4.30. Fresnel's theory of rotation of plane of polarisation :

Fresnel's theory is based on the simple principle that a linear simple harmonic motion can be assumed to be the superposition of two equal but opposite circular motions of half the amplitude, as has been proved in the section *Sound* in volume I of this book. Following the above principle, Fresnel assumed the following :

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* For Burdwan University only.

(i) A plane-polarised beam of light incident parallel to the optic axis of a crystal, is split up into equal and opposite circularly polarised beam when it enters into the crystal. One of the circularly polarised beams is right-handed and the other is left-handed.

(ii) In an optically *inactive* crystal like calcite etc., both the circularly polarised beams travel with same angular velocity and on emergence from the crystal, produce a plane-polarised beam again without any change in the plane of polarisation.

(iii) In an *optically active* crystal like quartz, etc., the two circularly polarised beams travel with different velocities. If the crystal is dextro-rotatory, the right-handed circularly polarised beam travels faster than the left-handed component while in a laevo-rotatory crystal, the left-handed component travels faster than the right-handed component. As a result, when they come out of the crystal, they produce again a plane-polarised beam but the plane of polarisation of the emergent beam rotates clockwise in the first case but anti-clockwise in the second case.

Suppose, a plane-polarised beam is incident on a crystal along its optic axis YOY'. When it enters into the crystal, it will be split up into two equal and opposite circular vibrations. If P_1 and P_2 be the generating points of those two circular vibrations, then in a crystal like calcite, their velocities will be equal and opposite. If P_1 and P_2 are supposed to start simultaneously from Y in the opposite directions, [Fig. 4.41] then it is easy to understand that in the circumstances, their resultant will be along YOY'. Consequently when the circular vibrations come out of the crystal, they give rise to a linear vibration along YOY' *i.e.*, a plane-polarised light with no change in the plane of polarisation. Hence, calcite crystal cannot cause any optical activity.

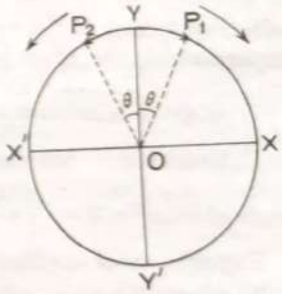

Fig. 4.41

But if the crystal be leavo-rotatory, then the component P_2 moves faster than the component P_1. If the generating points be supposed to start simultaneously from Y in the opposite directions, then in the time when P_2 makes a complete rotation and arrives at Y again, the other generating point P_1 will lag behind and may reach the point P_1 as shown in the fig. 4.42. Hence, there will be a phase-difference between the two and the phase angle is $\angle P_1OP_2 = \delta$(say).

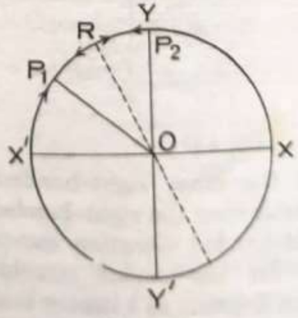

Fig. 4.42

When the circular vibrations come out of the crystal they travel in the same direction and hence produce a plane-polarised beam again but the vibrations of the plane-polarised beam take place along the direction OR. Hence, the rotation of the plane of polarisation.

$$\theta = \angle YOR = \frac{\delta}{2}.$$

Relation between angle of rotation and thickness of the crystal :

The angle of rotation of the plane-polarisation may be expressed in terms of the thickness of the crystal in the following way.

Let x be the thickness of the plate and the velocities of right-handed and left-handed circular vibrations be V_R and V_L respectively ($V_L > V_R$). Now, the difference of time taken by the two components to cross the thickness of the crystal plate $= \frac{x}{V_R} - \frac{x}{V_L} = x\left(\frac{1}{V_R} - \frac{1}{V_L}\right)$.

Hence, the phase-difference $\delta = \frac{2\pi}{T}.x\left(\frac{1}{V_R} - \frac{1}{V_L}\right)$ where T is the time-period of light wave in air.

If λ and V be the wavelength and velocity of light wave respectively in air,

then, $T = \frac{\lambda}{V}$;

hence, $\delta = \frac{2\pi}{\lambda}.x\left(\frac{V}{V_R} - \frac{V}{V_L}\right) = \frac{2\pi x}{\lambda}$ $(\mu_R - \mu_L)$, where μ_R and μ_L are the refractive indices of the crystal for right-handed and left-handed vibrations respectively.

$\therefore$ Angle of rotation $\theta = \frac{\delta}{2} = \frac{\pi x}{\lambda}$ $(\mu_R - \mu_L)$ ... ... (i)

If the crystal be however, dextro-rotatory, then $V_R > V_L$. In that case, the angle of rotation $\theta = \frac{\delta}{2} = \frac{\pi x}{\lambda}(\mu_L - \mu_R)$.

Experimental verification of the theory :

To establish his theory Fresnel used a combination of several right-handed and left-handed quartz prisms as shown in [Fig. 4.43]. The edges of the prisms at the two extreme ends are suitably cleaved to give the combination a rectangular shape. R and L represent the right-handed and left-handed quartz crystals respectively. They are put side by side in the combination. We know that when a plane-polarised beam enters a quartz prism, it is broken up into two equal and opposite circularly polarised beams—one left-handed and the other right-handed. Further, the left-handed circular vibrations move faster than the right-handed vibrations in a left-handed quartz while the right-handed vibration moves faster in a right-handed quartz. Consequently, for left-handed circular vibration, L-prism will behave as a rarer medium and R-prism as a denser one. On the other hand, for a right-handed circular vibration, L-prism will behave as a denser medium and R-prism as a rarer one. Also we know that when a ray, coming from a rarer medium is refracted through a prism of denser medium, the ray bends towards the base of the prism.

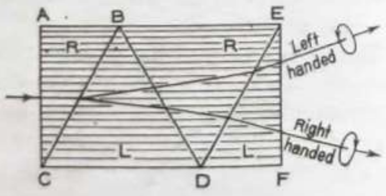

Fig. 4.43

Let us see what happens if we apply the aforesaid principle to the above combination of L and R quartz prisms. The prisms are so cut from the quartz crystals that their optic axes are perpendicular to AC or EF. In other words, the optic axes of the prisms are parallel to the base CDF. Consider now a plane-polarised light incident normally on the surface AC of the first prism. When the beam enters the prism, it is broken into two circularly polarised beams which travel in the same direction with different velocities. As the two circularly polarised beams enter the second prism, the right-handed vibration is deviated towards the base CD and the left-handed vibration in the opposite direction because the prism BCD being left-handed is denser towards the right-handed vibration but rarer towards the left-handed vibration. When the beams pass through the prisms BDE and EDF in succession, their separation gradually increases due to the above reason. As a result, if Fresnel's theory is true, we should get two circular vibrations after emergence from the prism-combiantion—one anti-clockwise and the other clockwise. Analysing the emergent beam with the help of a quarter-wave plate it has been experimentally found that the emergent beam actually consists of two circularly polarised beams with opposite vibrations. Thus, Fresnel's theory of optical activity is experimentally verified.

□ *4.31. Some optical phenomenon involving light, magnetism and electricity :

Since light is an electromagnetic wave, electric field and magnetic field have significant effects on light. Faraday, Zeeman, Kerr and others investigating into these phenomena, discovered several important magneto-optical and electro-optical effects.

(a) Faraday effect :

When a plane polarised beam passes through a glass block, no change takes place in its plane of polarisation because under ordinary circumstances glass is not optically active. But Michael Faraday in 1864, noticed that if the glass block be kept in a strong magnetic field, the block temporarily acquires the property of optical activity and rotates the plane of polarisation of plane polarised light. This phenomenon is known as *Faraday effect.*

Consider an unpolarised beam incident normally on a nicol prism P_1. When it emerges, it becomes plane-polarised with vibrations parallel to the optic axis of the nicol P_1. The beam is then allowed to pass through a hole bored along the axis of an electromagnet ($M - M$) and then falls on the analysing nicol P_2.

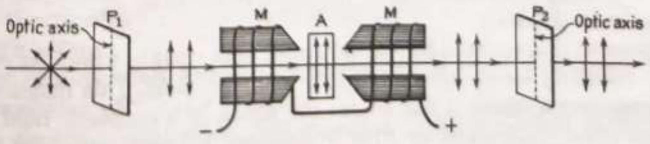

Fig. 4.44

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* For North Bengal and Kalyani Universities only.

A glass block A has been kept in the space between the pole pieces of the electromagnet. If no current flows through the electromagnet (*i.e.*, without applying the magnetic field) and the optic axes of the crystals P_1 and P_2 are parallel to each other, light, in its full strength, will emerge from the analysing nicol P_2 [Fig. 4.44]. Now rotate the analyser and keep it in a crossed position with respect to the polarising nicol P_1. In this position, no light will emerge from the nicol P_2. If, now the magnetic field is applied on the glass block A by sending strong current through the electromagnet, it will be seen that some light is coming out of the analyser P_2 although it is still in the crossed position. This indicates that the plane of polarisation of the plane polarised light has undergone some rotation. If the analyser P_2 be further rotated so as to block the light completely, the rotation of the analyser gives the angle through which the plane of polarisation has turned. It is found that this angle depends on (i) the intensity of the magnetic field and (ii) the length of the glass block traversed by light. If θ be the angle of rotation of the plane of polarisation, then,

$$\theta = V.H.l.$$

Here, H = intensity of magnetic field and l = length of the glass block traversed by light and V = a constant. The constant is however, known as *Verdet's constant.*

Characteristics of Faraday effect :

(i) The rotation of plane of polarisation is found to be the greatest when light travels along the lines of force of the magnetic field and the least or zero when light travels perpendicular to the lines of force of the magnetic field.

(ii) If the current flows in a clockwise direction through the electromagnet, the rotation of plane of polarisation is also clockwise or right–handed. If the current through the electromagnet is anti–clockwise, the rotation is also anti–clockwise or left–handed.

(iii) If the beam of light goes through the block of glass one way and is reflected back the other way through the block, the rotation of plane of polarisation is doubled. In the case of optically active substances, the direction of rotation is reversed with the reversal of the beam of light with the result that rotation for one way of light is cancelled by the opposite rotations for the reflected way. This is the main difference between ordinary optical activity and optical activity induced by magnetic field. For this reason, if a beam of light be sent to and fro along the direction of lines of force through the glass block the rotation is greatly amplified due to repeated reflections.

(b) Zeeman effect :

The spectral line emitted by excited atoms split up into a doublet or a triplet when the emitting atoms are placed in a magnetic field. This magneto–optical effect of splitting a spectral line under the action of a magnetic field is known as *Zeeman effect* discovered by Prof. P. Zeeman, a Dutch Physicist in1896.

As an example, let us subject a sodium source to a strong magnetic field and examine the spectral line by a high resolution spectroscope. It is observed that each spectral line is split into two or more components. If light is examined at right angles to the direction of the magnetic field, then the original line is split into three components. One of these line is in the same position as the original line and the other two lines are on the two sides of the original line. The two outer lines, when examined by means of a nicol prism as an analyser are found to be polarised at right angles to the original line. The effect is known as *transverse Zeeman effect* [Fig. 4.45(i)].

If observations are taken along the direction of magnetic field by boreing an axial hole in the pole–pieces of the electromagnet, there is no line in the position of the original spectral line and only two outer lines are present. The lines are found to be circularly polarised in opposite directions [Fig. 4.45(ii)]. This effect is termed as *longitudial zeeman effect.*

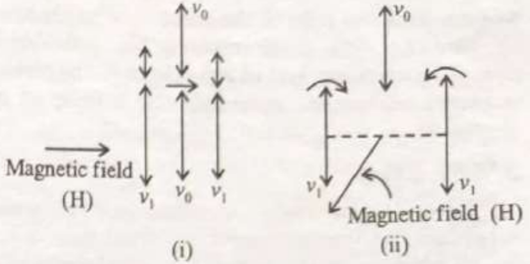


Fig. 4.45

The above effects are observed under a strong magnetic field and are generally known as *normal zeeman effect.* If weak magnetic field is used to split the spectra lines, many other lines are found to be present and the effect is then termed as *anomalous zeeman effect.*

Fig. 4.46 shows an outfit for observing zeeman effect in a laboratory. The normal zeeman effect is explained on Lorentz electron theory or more conveniently by quantum mechanics.

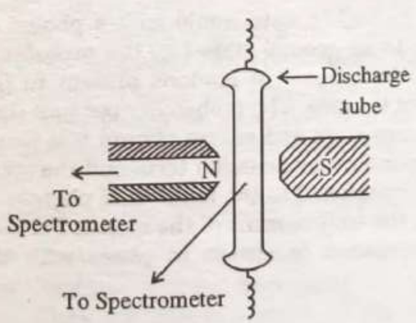


Fig. 4.46

(c) Stark effect :

If instead of magnetic field, the spectral liens are subjected to strong electric field, similar splitting of lines occur. This phenomenon is known as *stark effect* after the name of its discoverer J. Stark of Germany in 1913. Like zeeman effect it is also classified into *transverse* and *longitudinal* effects depending upon whether the direction of observation is at right angles to or parallel to the direction of the applied electric field. Quantum theory offers a satisfactory explanation of Stark effect.

(d) Kerr effect :

Mention of Kerr effect has already been made in art. 1.3 in connection with the determination of velocity of light.

It is an electro–optical effect, first observed by John Kerr is 1876. Normally glass is isotropic, but when a piece of glass is placed between the plates of a capacitor on which an electric field is applied, the piece becomes doubly refracting. A ray of light passing through the piece splits up into E–ray and O–ray which travel with different velocities. One of the components is polarised with its plane of vibration along the direction of the electric field, whereas the other component has its plane of vibration perpendicular to the electric field.

The two components traversing an electric field of length l and of strength E have a path–difference of n wavelength introduced between them which is given by $n = KlE^2$ where K is the *Kerr constant* for the substance used. One of the common substances showing strong Kerr effect is nitro–benzene for which the value of K is 2.7×10^{-8}. A capacitor having nitro–benzene placed between its plates forms a *Kerr cell* (Fig. 1.5). Such a cell placed between two crossed nicols will not pass any light in the absence of an electric field. If the field is switched on, nitro–benzene starts behaving like a doubly refracting crystal and hence passes a certain amount of light through the nicols. This property of a kerr cell is used to determine accurately the velocity of light. (See Kerr cell method, chaper 1).

❑ *4.32. Laser :

"Laser" means *L*ight *A*mplification by *S*timulated *E*mission of *R*adiation'. It produces an intense beam of coherent light (*i.e.*, the wave motion is in phase for all photons in the beam) the wavelength of which corresponding to one particular energy–level transition for a large number of atoms.

According to quantum theory of radiation, an atom may undergo transition between energy states if it emits or absorbs a photon of the appropriate energy. When an atom in the ground state, for exmaple, absorbs a photon and makes a transition to a higher energy state or level, the photon is said to have 'stimulated' the absorption. The atom cannot increase its energy 'spontaneously' *i.e.*, in the absence of a photon.

We might expect that the atom in its higher state would emit a photon of energy $h\nu$ spontaneously in returning to its ground state *i.e.*, the probability of emission would be independent of the number of photons present in the environment of the atom. But that is not the case. The probability per unit time that an atom will decay to a lower energy state and emit a photon was found to be the sum of two terms. One is a spontaneous emission term and the other is a '*stimulated term*' which is again proportional to the number of photons of the relevant energy already present in the environment of the atoms. Further, the photon produced by stimulated emission is always in phase with the stimulating photon.

Einstein's A & B coefficient's :

Consider an atom having two states [Fig. 4.47]. N_1 and N_2 are the number of atoms per unit volume in states 1 and 2 respectively. Let $u(\nu)$ represent the energy density of radiation *i.e.*, $u(\nu)d\nu$ represents the energy density in the frequency interval between ν and $\nu + d\nu$. Now absorbing radiation of frequency ν, an atom will jump from the state 1 to state 2.

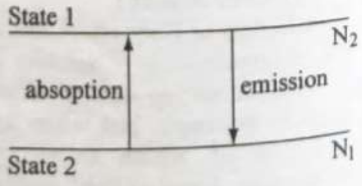


Fig. 4.47

The excited atom will now emit photon and will come down from the state 2 to the original state 1. This can happen in two ways *viz.*, (i) spontaneous emission and (ii) stimulated emission. The number of spontaneous emission will be proportional to N_2 and if the proportionality constant is denoted by A, then N_2A will represent the number of spontaneous emissions per unit volume per unit time. Finally the number of stimulated emission will be proportion to N_2 and also to the radiation energy density $u(\nu)$. If the proportionality constant is denoted by B, then $N_2B\ u(\nu)$ will represent the number of stimulated emissions per unit volume per unit time. The quantities A and B are known as Einstein's coefficients. It may proved that

$$\frac{8\pi h\nu^3}{C^3}.B = A.$$

The above equation gives the relation between the Einstein's coefficients.

General principle of laser :

Let us consider the case of an atom with energy level E above the ground state of energy E_0, for which the probability of spontaneous emission is nearly nil but that of stimulated emission is fairly high. Suppose, that somehow a large number of atoms are injected into this excited state E. As the stimulated emission term is proportional to the number of photons present in the neighbourhood of the excited atoms, these atoms will not decay until there are photons of energy $(E - E_0)$ present in the system.

If, now, a few photons of the requisite energy (*i.e.*, $E - E_0$) are introduced, stimulated emission will take place immediately and a number of photons of the same kind will be available. This increases the number of photons, which in turn, stimulate the emission of more photons. Thus, a 'chain–reaction' is produced with the result that all the stoms present emit their photons very rapidly. This process is called 'light amplification by stimulated emission of radiation', since the original pulse of photons has been amplified into a much more powerful pulse. The word *LASER* has been coined by putting the initial words of the name of the above process.

The laser pulses have the following characteristics :

(i) They are *monochromatic* because all the photons possess the same energy $(E - E_0)$ above the ground state and hence the same wavelength $\lambda = \frac{c.h}{E - E_0}$, where c = velocity of light and h = Planck's constant.

(ii) They are *coherent* because all the waves are in same phase.

(iii) They are *intense* because all the waves are coherent. Had the waves been out of phase or non–coherent, the resultant intensity would have been proportional to $n \times a^2$ where a is the amplitude of each wave and n is the number of waves. As the waves are in phase, however, their total amplitude becomes $n.a$ and hence the intensity becomes proportional to $n^2.a^2$ *i.e.*, the intensity becomes greater by a factor n than that obtained from non–coherent waves. Since n is very large, the intensity is increased enormously.

(iv) They are *rectilinear* and *almost parallel* so much so that a laser beam sent from a tube of 30 cm diameter will have a divergence less than a kilometer when it arrives at the surface of the moon.

Principle of Ruby laser :

Laser may be of three types : (i) ruby lasers (ii) gas lasers (iii) lasers made of semi–conductors. The means of 'pumping' and of stimulating are different in three cases. We consider here the ruby laser whose principle is as follows :

Ordinarily, in a system of atoms in thermal equilibrium with some kind of electromagnetic radiation, the distribution of energies, according to Boltzmann law is given by :

$$\frac{n_2}{n_1} = e^{(E_1 - E_2)Kt}$$

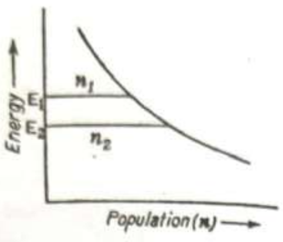


Fig. 4.48

where n_2/n_1 is the ratio of the numbers of atoms having energies E_2 and E_1 at a temperature T, k being Boltzmann constant. Thus higher the energy of a level, the lower is the number of atoms in it (Fig. 4.48). This is called a *'normal population'* of atoms among the available energy states. It is the reversal of this law in certain circumstances that is referred to as *'population inversion'*. In a ruby laser population inversion is produced in the following way :

Consider an atom which has three energy levels E_0 (ground state), E_1 and E_2' (Fig. 4.49). By means of a flash tube, for example, a large number of atoms is first excited or 'pumped' to an energy state E_2 by photons of energy $(E_2 - E_0)$ shown by the stage no. 1. The excited atoms undergo spontaneous decay to the lower level E_1 emitting photons of energy $(E_2 - E_1)$ as shown by the stage no. 2.

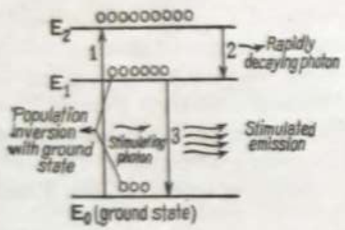


Fig. 4.49

Now, the energy state E_1 acquires the special property of having a large probability of stimulated emission so that atoms of this energy state are unable to decay spontaneously. The energy state E_1 is thus surrounded with a far greater number of atoms than the ground state E_0, thus bringing about a population inversion between these two energy states.

Any stray photon with the requisite energy $(E_1 - E_0)$ will now produce stimulated emission, followed by chain reaction as described earlier.

Description of Ruby laser : Fig. 4.50 shows the form of a ruby laser tube. Ruby is alumina (Al_2O_3) with about 0.05 per cent of the aluminium atoms replaced by chromium. It is only the chromium atoms which take part in the laser action. The tube consists of a hollow cylinder of light-transparent ruby, one face of which is covered by a fully silvered mirror and the other by a partially silvered mirror. To stimulate the atoms, a xenon flash-tube is wound like a cork-screw over the cylinder. When the flash tube operates above a certain critical light intensity, a population inversion is created among the chromium atoms. They are first excited to the energy level E_2 (fig. 4.50) where they stay for a very short while to the extent of about 10^{-8} s and drop into the level E_1 which is metastable. This means that a transition to the ground state is very imporable, so that they remain in it long enough for most of them to be triggered and emit stimulated rather than random radiation.

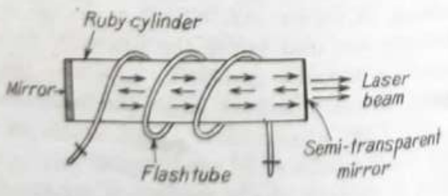


Fig. 4.50

The function of the mirrors is to reflect the stimulated radiation backwards and forwards from ends of the cylinder and generate more coherent photons from the population all moving in one direction. A narrow intense flash emerges from the partially silvered end, lasting for about a microsecond. Several flashes occur per millisecond when the xenon flash tube triggers as well as pumps. The output wave from a ruby laser has a wavelength λ = 6943A° with a spread of only 0.02A°.

Applications of laser include local melting, cutting and welding. A lens must be used to concentrate the beam on a very small spot in order to perform these operations. The transmission of modulated signals over very long distances can be achieved with a laser beam. Its highly directional property makes it a suitable device for use in rader. Scientists believe that laser will be a valuable tool for space research and investigations.

Gas (Helium-neon) laser : Laser action has been made possible in a mixture of inert gases like helium and neon and this has helped the scientists to develop gas laser which has some advantages over solid ruby laser. Fig. 4.51 shows the basic features of a gas laser tube. A mixture of helium and neon gases (sometimes carbon dioxide is also used) is taken inside a long quartz tube. The two ends of the tube are closed by two optically plane mirrors M and M. Discharge in the gas mixture is produced by a powerful radio-frequency generator so that helium atoms are excited or pumped up to higher energy level. The neon atoms are then excited to a higher energy level by collision with energised helium atoms and population inversion is created. Stimulated emission then occurs as the neon atoms undergo transition to a lower energy level. A He–Ne laser gives out radiation of wavelength 6328A° in optical region. However 1.15μm and 3.39μm wavelength are also provided by He-Ne laser in near infra-red and infra-red region respectively.

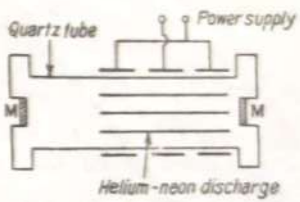


Fig. 4.51

A gas laser has several advantages over the ruby laser or the so-called solid laser ; an important advantage being that the light is produced as a continuous beam rather than in ultra-short pulses as in ruby laser. Further in ruby laser, the crystal and other imperfections in solid cause a slight divergence of the beam and slight spread of wavelengths. In gas laser this does not happen.

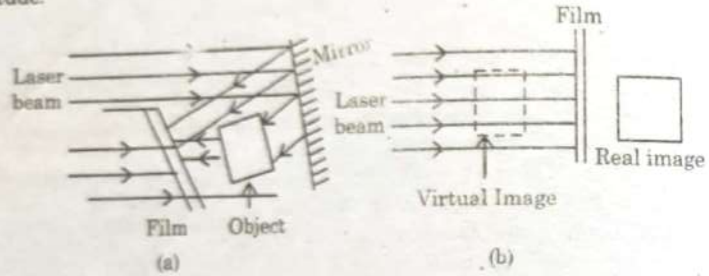
☐ 4.33. Laser versus ordinary light :

Laser	Ordinary light
1. The light pulse from a leser is strictly monochromatic because all the photons have the same energy above the ground level and hence the same frequency.	1. Ordinary light is not monochromatic. It is composed of seven different wavelengths of light.

Laser	Ordinary light
2. The light pulse from a laser is highly coherent because all the photons or waves are in phase.	2. Ordinary light is not coherent because the waves are not in phase.
3. Laser pulses are very intense.	3. Ordinary light is less intense.

☐ *4.34. Hologram :

The idea of the hologram was first proposed by Dr. D. Gabor in 1948 who named it from the Greek word *holos*, meaning whole. A hologram contains the whole of the information of the wave from the object phase as well as amplitude.



(a)

(b)

Fig. 4.52

Unlike a normal two–dimensional image formed on a piece of paper by a glass lens, a hologram provides a true three dimensional record of the object. The hologram itself is two dimonsional but using a laser a three dimensional image may be constructed. The hologram is formed by allowing a reference beam to interfere with the light scattered from the object, using a mirror and a photographic plate [Fig. 4.52].

The image of the object may be reconstructed from the hologram by allowing a laser beam to fall on it. Fig. 4.50(b) shows the process. If you move your head and view the image from different positions you will see that it is truely three dimensional.

❖ SOLVED PROBLEMS ❖

1. *Calculate the polarising angle for light travelling from water of refractive index 1.33 to glass of refractive index 1.53.*

Ans. According to Brewster's law, $\mu = \tan p$, where μ is the refractive index of the reflecting surface with respect to the medium through which light was coming.

In this case, $_{\omega}\mu_g = \tan p$ or, $\tan p = \frac{\mu_g}{\mu_w} = \frac{1.53}{1.33} = 1.15 = \tan 49°$

$$\therefore p = \mathbf{49°}.$$

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* For Burdwan University only.

2. *A polariser and an analyser are oriented so that maximum amount of light is transmitted. To what fraction of its maximum value is the intensity of the transmitted light reduced when the analyser is rotated through 60° ?*

Ans. Let I be the maximum intensity of the transmitted light. According to the law of Malus, $I_1 = I.\cos^2\theta = I.(\cos 60°)^2 = \frac{1}{4}I.$

$$\therefore \frac{I_1}{I} = \frac{1}{4} = \mathbf{0.25}.$$

3. *Calculate the thickness of a quartz half-wave plate for the line 6563A° for which the extraordinary and ordinary refractive indices are $\mu_e = 1.55085$ and $\mu_0 = 1.54184$.* **[C.U. 1997]**

Ans. Since quartz is a positive crystal $\mu_e > \mu_0$; so if t be the thickness of the half-wave plate,

$$t = \frac{\lambda}{2(\mu_e - \mu_0)} = \frac{6563 \times 10^{-8}}{2(1.55085 - 1.54184)} = \frac{6563 \times 10^{-8}}{2 \times 0.00901} = \mathbf{3.64 \times 10^{-3}\ cm.}$$

4. *Calculate the thickness of quarter-wave plate of quartz with $\lambda = 5.8 \times 10^{-7}$ m; $\mu_e = 1.553$ and $\mu_0 = 1.544$.* **[C.U. 1985]**

Ans. For a quarter-wave plate of quartz, we have,

$$t = \frac{\lambda}{4(\mu_e - \mu_0)} = \frac{5.2 \times 10^{-7}}{4(1.553 - 1.544)} = \frac{5.9 \times 10^{-7}}{4 \times 0.009} = \mathbf{1.64 \times 10^{-5}\ m.}$$

5. *A beam of plane–polarised light is changed into circularly polarised light by passing it through a slice of crystal 0.0028 cm thick. Calculate the difference in refractive indices of the two rays in the crystal, assuming this to be the minimum thickness that produces the effect. The wavelength of light used $\lambda = 5.8 \times 10^{-7}$ m.*

Ans. For conversion of plane–polarised light into circularly polarised light, the least phase–difference between E-ray and O-ray should be $\pi/2$. Hence, the crystal plate must be a quarter-wave plate (See art. 4.16), for which,

$$\text{or, } t = \frac{\lambda}{4(\mu_e - \mu_0)} \quad = (\mu_e - \mu_0) = \frac{\lambda}{4t} = \frac{5.8 \times 10^{-7}}{4 \times 0.0028 \times 10^{-2}} = \mathbf{0.52 \times 10^{-2}}.$$

6. *Plane–polarised light is incident normally on a piece of quartz cut parallel to the axis. Find the least thickness for which the ordinary and the extra-ordinary rays combine to form plane polarised light again. Given $\mu_0 = 1.5442$; $\mu_e = 1.5533$ and $\lambda = 5 \times 10^{-7}$ m.*

Ans. It is easy to understand that the quartz plate should be a half-wave plate ; because the phase-difference introduced between the E-ray and the O-ray by half-wave plate $= \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$; further, since their planes of vibrations are mutually perpendicular and they travel along the same line, they will produce a plane–polarised beam on emergence.

Now, the thickness of a half-wave plate

$$t = \frac{\lambda}{2(\mu_e - \mu_0)} = \frac{5 \times 10^{-7}}{2(1.5533 - 1.5442)} = \mathbf{2.75 \times 10^{-5}\ m.}$$

7. *A tube 20 cm long filled with a solution cane sugar placed in the path of a polarised light, gives an optical rotation of 11°. Find the strength of the solution if the specific rotation of cane sugar is 66°.* **[N.B.U. 2001]**

Ans. We know, $[S]_t^\lambda = \frac{10\theta}{l.c.}$, Here, $[S]_t^\lambda = 66°$; $l = 20$ cm ; $\theta = 11°$

$$C = \frac{10 \times 11}{20 \times 66} = \frac{1}{12} = 0.0833$$

$\therefore$ **C = 8.33%.**

8. *The indices of refraction of quartz for right–handed and left–handed circularly polarised light of wavelength 7620A° are 1.53914 and 1.53920 respectively. Calculate the rotation of the plane of polarisation of the light in degrees produced by a plate of 0.5 mm thickness.*

Ans. The rotation of plane of polarisation is given by, $\theta = \frac{\pi x}{\lambda}(\mu_L - \mu_R)$ for a dextro–rotatory substances (art. 4.29).

According to the problem, $x = 0.5$ mm $= 0.05$ cm ; $(\mu_L - \mu_R) = 1.53920 - 1.53914) = 0.00006$ and $\lambda = 7620A° = 7620 \times 10^{-8}$ cm.

$$\text{Putting these values, } \theta = \frac{\pi \times 0.05(0.00006)}{7620 \times 10^{8}} \text{ rad}$$

$$= \frac{\pi \times 30}{762} \text{ rad} = \frac{\pi \times 30}{762} \times \frac{180°}{\pi} = \mathbf{7.1°}.$$

9. *The rotation of the plane of polarisation (λ = 5896 A°) in a certain substance is 10° per cm. Calculate the difference between the refractive indices for right and left circularly polarised beams in the substance.*

Ans. We know, the angle of roation δ is given by,

$\delta = \frac{\pi . x}{\lambda}(\mu_R - \mu_L)$ [eqn. (i) art. 4.29]

or, $\frac{\delta}{x} = \frac{\pi}{\lambda}(\mu_R - \mu_L)$; here $\frac{\delta}{x} = 10° = \frac{10 \times 2\pi}{360}$ radian.

$$\therefore \frac{10 \times 2\pi}{360} = \frac{\pi(\mu_R - \mu_L)}{5896 \times 10^{-8}}$$

$$\text{or, } (\mu_R - \mu_L) = \frac{5896 \times 10^{-8} \times 10 \times 2}{360} = \mathbf{3.27 \times 10^{-6}}.$$

10. *A quartz plate is cut with its surface perpendicular to the optic axis and is required to annul completely the rotation of the plane of polarisation of red light (λ = 7600A°) by a 26.7 cm length of lactose solution containing 100 g of active substance per litre of the solution. What should be the thickness of the plate? Sp. rotation of lactose = 52.5° ; for quartz plate with light of λ = 7600 A°, $\mu_L = 1.53920$ and $\mu_R = 1.53914$.*

Ans. For an optically active solution, we have, $[S]_t^\lambda = \frac{\theta}{l.c}$;

Here, $[S]_t^\lambda = 52.5°$; $l = 26.7$ cm $= 2.67$ decimeter ;

$$c = \frac{100}{1000} = 0.1 \text{ g/cm}^3.$$

$$\therefore \theta = [S]_t^\lambda \times l \times c = 52.5 \times 2.67 \times 0.1 = 14.02°.$$

Again, for a quartz (dextro-rotatory) plate, the angle of rotation,

$$= \frac{\pi x}{\lambda}.(\mu_L - \mu_R) \text{ radians} = \frac{\pi x}{\lambda}.(\mu_L - \mu_g) \times \frac{180}{\pi} \text{ degrees}$$

$$= \frac{x}{7600 \times 10^{-8}}(1.53920 - 1.53914) \times 180 = \frac{2700 \times x}{19}$$

For complete annulment, $\frac{2700 \times x}{19} = 14.02$

or, x = **0.099 cm.** (nearly)

11. *A certain length of 5% solution causes an optical rotation of 20°. How much length of 20% solution of the same substance will cause a rotation of 35° ?*

Ans. Since same solution is used, the specific rotation will remain the same. In this case, we have,

$$\frac{\theta_1}{l_1 c_1} = \frac{\theta_2}{l_2 c_2} \text{ or, } \frac{20}{l_1 \times 5} = \frac{35}{l_2 \times 20} \text{ or, } l_2 = \frac{l_1 \times 35 \times 5}{20 \times 20}$$

$$\therefore l_2 = \mathbf{0.438\ l_1} \text{ (nearly).}$$

◆ QUESTIONS ◆

○ *Essay type :*

1. Write a short essay on polarisation of light.

2. Describe a method of polarising a beam of light by reflection. **[C.U. 1920 ; N.B.U. 2001]**

3. State Brewster's law. Show that the angles of incidence and refraction are complementary when maximum polarisation is obtained by reflection at a plane glass surface.

4. Explain the principle and working of Biot's polariscope. What is Malus' law ? **[N.B.U. 1982 ; C.U. 2004]**

5. Describe the construction and action of a nicol prism. **[C.U. 1963,'67,'87]**

6. Describe the construction of a nicol prism and explain how it acts as (a) a polariser and (b) an analyser. **[C.U. 1999 ; Burd. U. 2004, Tripura. 1990]**

7. Give an account of Huygens theory of double refraction in a uniaxial crystal. What are positive and negative crystals ? Name a crystal of each type.

8. Enumerate the steps you would follow to investigate qualitative a beam of light for polarisation characteristics. The beam may contain (a) unpolarised and plane–polarised light (b) unpolarised and elliptically polarised light (c) circularly polarised light only.

9. Describe a half-shade polarimeter and explain the action of its optical parts.

10. (a) Describe an arrangement for determining accurately the optical activity of liquids. To what practical uses does its study lead ?

(b) What is optical activity ? State with example the classification of optically active substances. **[C.U. 2001]**

11. Show that a beam of plane–polarised light can be regarded as being composed of two equal and opposite circularly polarised lights.

12. Describe Laurent's half-shade polarimeter. In what respect is a bi-quartz better than a half-shade plate ?

13. What is Babinet's compensator ? Explain how this can be used to produce (i) circularly polarised (ii) elliptically polarised beams.

14. What is Laser? Explain, in brief, the general principle of Laser.

○ ***Short answer type :***

15. Explain the terms: (i) polarised light (ii) plane of polarisation (iii) Extra-ordinary ray (iv) Optic axis. **[C.U. 1985]**

16. What is plane of polarisation and in which of the types of waves – longitudinal and transverse – can it occur?

17. State Brewster's law. **[C.U. 1999]** Prove that when light strikes a plane parallel glass plate at the polarising angle, refracted beam also falls on the second surface at its polarising angle.

18. Distinguish between polarised and unpolarised light. **[N.B.U. 2001]**

19. What is doubly refracting crystal? What is the difference between ordinary and extraordinary rays? Distinguish between positive and negative crystals.

20. Two polarising sheets initially have their polarisation directions parallel. Through what angle must one sheet be turned so that the intensity of transmitted light is reduced to a third of the original intensity? **[C.U. 1999]**

21. What are half-wave and quarter-wave plates?

22. What is elliptically polarised light? Can its transmission be prevented by an analyser?

23. What is optical activity? Define specific rotation for both solids and solution. **[C.U. 2001, '03]**

24. What is a half-shade plate? What is the difference between a half-shade plate and a biquartz plate?

25. What are polariods? Mention some of the uses of polaroids.

26. A beam of light is incident on a quarter-wave plate. The emergent light is viewed through a rotating nicol. What change in the intensity of light will be noticed if (i) the incident light is unpolarised (ii) the incident light is polarised with vibrations making an angle 45° with the axis of the plate (iii) the incident light is polarised with vibrations inclined at any angle with the optic axis?

27. (a) What are the wavelength of radiation emitted by ruby laser and He-Ne laser. **[Burd. U. 2004]**

(b) What are Einstein's A & B coefficients? What is the relation between them?

28. Write notes on (a) Faraday effect (b) Zeeman effect (c) Kerr effect. What do you mean by Raman effect? **[C.U. 2001]**

○ ***Numerical Problems :***

29. (a) Calculate the polarising angle for diamond surface, if the angle of refraction of a beam of light through it is 12° when the incident beam makes an angle of 60°. [Ans. 76.5]

[**Hints** : $\mu = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 12°} = \frac{0.8660}{0.2079} = 4.16$. Now apply the formula, $\tan p = \mu$]

(b) Refractive index of glass is 1.5. Calculate the Brewster's angle for it. Also calculate the angle of refraction. **[N.B.U. 2000]**[Ans. 56° ; 33°30′]

30. (a) It is found that when light is incident on a glass block, the reflected beam is completely plane polarised, when the angle of incidence is 57°. What is the refractive index of glass? [Ans. 1.54]

(b) The angle of refraction of an unpolarised beam of light, incident at a polarising angle on a glass block is 32.5°. Calculate the refractive index of glass. **[K.U. 2003]** [Ans. 1.57]

(c) Light reflected from a smooth ice surface, is found to be completely polarised. Find the angle of incidence of light, if the refractive index of ice is 1.309. **[C.U. 2005]** [Ans. 52°36′(approx.)]

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31. If the polarising angle of a piece of glass for red light is 60°, find the angle of minimum deviation for an equilateral prism of same glass for the same light. [Ans. 21°50′]

32. Determine the thickness of a quarter-waveplate for light of wavelength 5893 A° for which the refractive index for ordinary ray is 1.54 and for extraordinary ray 1.55. [Ans. 1.47×10^{-3} cm]

33. Unpolarised light is incident normally on a piece of quartz plate cut parallel to its principal axis. After emergence the path-difference between ordinary and extra-ordinary ray is $\frac{\lambda}{2}$. Find the thickness of the plate. Given $\mu_0 = 1.54442$ and $\mu_e = 1.5533$ and $\lambda = 5000$ A°. **[C.U. 1993]** [Ans. 2.74×10^{-3} cm]

[**Hints :** The plate is a half-wave plate]

34. Plane-polarised light is incident on a piece of quartz cut parallel to the axis. Find the least thickness for which the ordinary and the extraordinary rays combine to form plane polarised light given that $\mu_0 = 1.5442$ and $\mu_e = 1.5533$, $\lambda = 5 \times 10^{-5}$ cm. [Ans. 2.74×10^{-3} cm]

35. A plane-polarised beam is incident normally on a Babinet's compensator. The difference of the thickness of the two crystals of the compensator traversed by a light is 0.0016 cm. Calculate the phase-difference between the *E* and *O*-rays on emergence if the wavelength of the incident light is 5824°. Give $\mu_e = 1.5533$ and $\mu_0 = 1.5442$. [Ans. $\pi/2$]

[**Hints :** Use the formula ; Phase-diff. $= \frac{4\pi}{\lambda}(\mu_e - \mu_0)(t_2 - t_1)$]

36. An unknown solution is suspected to contain sucrose and does not contain any other optically active substance. If a 20 cm length of this solution rotates sodium light through 1°, what is the concentration of the source? Specific rotation of sucrose is 66°. [Ans. 0.0075 g/cm^3.]