2022

PHYSICS — HONOURS — PRACTICAL

Paper : CC-12P

(Statistical Physics)

Full Marks : 30

The figures in the margin indicate full marks.

Set-I

LNB	:	05
viva voce	:	05
Experiment	:	20

Answer ***any one*** question.

1. (a) With the help of appropriate *NumPy function* draw random samples (of size 10000) from a normal (Gaussian) distribution and plot the distribution in histogram. Given that the mean and the standard deviation of the distribution are 0 and 0.1 respectively.

   (b) A radioactive sample of Cobalt-60 initially contains 10000 nuclei. The half-life of Cobalt-60 is 5.27 years. Calculate statistically the number of nuclei (N) of Cobalt-60 present in the sample after a time t years. Plot the numerical solution i.e., N(t) vs t graph, with the exact analytical solution on the same graph. 5+15

2. (a) A random walker performs a one-dimensional random walk from origin. Draw a trajectory of the walker for 100 random steps.

   (b) Using central limit theorem (CLT) generate a Gaussian distribution from the uniform distribution [0,1). Determine the values of mean (μ) and standard deviation (σ) of the Gaussian distribution so obtained.

   Plot the histogram of this distribution, along with the probability density function

$$p(x)=\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$ 5+15

3. (a) A random walker performs a two-dimensional random walk starting from origin. Draw a trajectory of the walker for 1000 random steps.

(b) Using appropriate transformation generate an exponential distribution from the uniform distribution [0,1). Plot the histogram of the exponential distribution so obtained. With the help *of NumPy exponential function* generate a random sample having the same values of scale parameter and sample size and plot the histogram along with the earlier one. 5+15

4. (a) With the help of appropriate *NumPy function* draw random samples (of size 10000) from an exponential distribution and plot the distribution in histogram. Given that the mean of the distribution, $\beta = 5.0$.

(b) Evaluate $\int_0^1 \frac{x}{x^2+1}\,dx$ using Monte-Carlo Integration method. Plot the function and the points accumulated under the curve in this method. 5+15

5. (a) With the help of appropriate *NumPy function* draw random samples (of size 10000) from a normal (Gaussian) distribution and plot the distribution in histogram. Given that the mean and the standard deviation of the distribution are 0.5 and 0.2 respectively.

(b) Simulate an unbiased random walk in one-dimension and plot the RMS value of end-to-end distance as a function of time step (you may use a log-log scale). Fit the data and show that it follows a power law. Hence find the exponent of the power law. 5+15

6. (a) Plot the following distribution functions with energy for two temperatures, 20K and 400K:

(i) Bose-Einstein distribution : $f(E)=\frac{1}{e^{(E-\mu)/kT}-1}$ with $\mu = 0\ eV$ and

(ii) Fermi-Dirac distribution : $f(E)=\frac{1}{e^{(E-E_F)/kT}+1}$ with $E_F = 1\ eV$ and

the Boltzmann constant $k = 8.617\times10^{-5}\ eV/K$.

(b) Evaluate the error function erf $(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt$ for the following four values of x : x = 0.25, 0.5,1.0 and 2.0. 5+15

7. (a) With the help of appropriate *NumPy function* draw random samples (of size 10000) from an exponential distribution and plot the distribution in histogram. Given that the mean of the distribution, $\beta = 1.0$.

(b) Simulate a two-dimensional uniform random walk and plot the RMS value of end-to-end distance (R) as a function of time step (t) (you may use a log-log scale). Fit the data and show that it follows the power law $R \sim t^{1/2}$. 5+15

8. (a) Plot molar specific heat of solids by comparing (i) Einstein model: $C_v = 3R\left(\frac{T_E}{T}\right)^2 \frac{e^{T_E/T}}{\left(e^{T_E/T}-1\right)^2}$ and (ii) Debye model: $C_v = 9R\left(\frac{T}{T_D}\right)^3 \int\limits_0^{T_D/T} \frac{x^4 e^x}{\left(e^x-1\right)^2}\,dx$ for a temperature range of 1 *to* T_D Kelvin. Take Einstein temperature $T_E = 174\,K$, Debye temperature $T_D = 225\,K$ and $R = 8.31\,J\,\text{mol}^{-1}\,K^{-1}$.

(b) A single trial of a random experiment is tossing an unbiased coin 100 times. Simulate an experiment consisting of such 1000 trials. Plot the total number of Heads obtained per trial in a histogram. 10+10

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Set-II

LNB	:	05
viva voce	:	05
Experiment	:	20

1. (a) Generate N uniform random numbers between 0 and 1. Store them in an array. Evaluate the mean and standard deviation of the numbers generated. Calculate the autocorrelation function $r_k = \frac{c_k}{c_0}$ of the series for $k = 2$ and $k = 10$. Print the values of mean, standard deviation and the values of r_2 and r_{10} of the random series.

(b) Plot Fermi Dirac Distribution $f(E) = \frac{1}{e^{\frac{E-E_f}{kT}}+1}$ vs. $\frac{E}{E_F}$. Given $E_F = 0.2\ eV$ and $k = 8.617\times10^{-5} eV/K$. Take the range of $\frac{E}{E_F}$ from 0 to 2. Take $T = 1K$, $T = 50\ K$ and $200\ K$. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

2. (a) Generate N uniform random number between 0 and 1. Store them in an array. Generate exponential distribution from the above array using a suitable transformation and store the data in another array. Find the mean m and standard deviation s of the exponential random numbers.

Compute k_3, the third order central moment about mean $\left(k_3 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3\right)$ for the created

exponential distribution. Define skewness as $\mu_3 = \frac{k_3}{s^3}$. Print the values of m, s, k_3 and μ_3.

(b) Plot the MB distribution $f(E)$ vs E, where $f(E) = \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{\frac{3}{2}}} \sqrt{E} e^{-\frac{E}{kT}}$, where $k = 8.617 \times 10^{-5} eV/K$.

Take the range of E from 0 to 0.5 eV. Take $T = 100\ K$, $T = 250\ K$ and $T = 500\ K$. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

3. (a) Generate N exponential random numbers (with mean 5) and store them in an array. Draw n samples from the array, each sample should contain s random numbers. Compute the mean of these n samples and store the means in another array. Plot the original array and the array of means in separate Histogram using subplot. Take $N = 100000$, $n = 1000$ and $s = 40$ as runtime inputs. Comment on the nature of the distribution that the means follow.

(b) Plot the BE distribution $f(E)$ vs E, where $f(E) = \frac{1}{e^{\frac{E-\mu}{kT}} - 1}$, $k = 8.617 \times 10^{-5}\ eV/K, \mu = 0.1 eV$. Take the range of E from 0.101 to 0.16 eV. Take $T = 10\ K$, $T = 50\ K$ and $T = 100\ K$. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

4. (a) A single trial of a random experiment is tossing a biased coin (p = probability of obtaining head in a toss) m times. Simulate an experiment consisting of N trials. Plot the total number of heads obtained per trial in histogram. Set a proper title. Save the graph in suitable format. (Take the values *of* $p = 0.2$, $m = 80$ and $N = 100$ during runtime.)

(b) Evaluate the following integral using Monte-Carlo method of integration. $\int_0^4 \frac{dx}{x^2 + 16}$. Take the number ($n$) of random points as input. Run the program for two different values of $n = 100$ and $n = 1000$. Print the results. 10+10

5. (a) Considering decay of a given nuclei to be independent of each other and $\lambda \equiv$ probability of decay of a single nuclei per unit time. Simulate radio active decay for N_0 number of parent nuclei at $t = 0$. Consider the final time to be t_n and number of un-decayed nuclei at time t_n be $N(t_n)$. Estimate the fraction of nuclei that remains at t_n. Also evaluate it directly using the standard relation and compare it with the results obtained from simulation. Use $\lambda = 0.005$, $N_0 = 60000$ and $t_n = 450$.

(b) The mean occupation number of a single particle state of energy ϵ is given by $\langle n_\epsilon \rangle = \dfrac{1}{e^{\frac{\epsilon-\mu}{kT}} + a}$

where $a = +1, -1$ for FD and BE distribution respectively. Plot $\langle n_\epsilon \rangle$ *vs.* $\dfrac{\epsilon-\mu}{kT}$ for the FD and BE distribution. Consider the range of $\dfrac{\epsilon-\mu}{kT}$ within –3 to 3. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

6. (a) A single 2D random walk consists of n steps. Simulate a collection of N such random walks. Evaluate the r.m.s. value of end to end distance (r) for different number of steps ((t) = 1 to n). Find the value of r at $t = 9$ and $t = 64$. Fit the following function with the data $r(t) = at^b$. Print the values of a and b. (Use the necessary scipy function for fitting)

(b) Given C_v according to Dulong Petit Law : $C_v = 3R$, and Einstein Theory : $C_v = 3R\dfrac{\left(\frac{\theta_E}{T}\right)^2 e^{\frac{\theta_E}{T}}}{\left(e^{\frac{\theta_E}{T}} - 1\right)^2}$

Assuming $\theta_E = 10\ K$, plot $\dfrac{C_v}{3R}$ vs T for T lying within the range 1 to 100K for Dulong Petit's Law and Einstein Theory. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

7. (a) Generate N normal random number with mean (μ) and standard deviation (σ). Store it in an array. Evaluate mean and standard deviation for a randomly selected sample of 20 numbers from the array. Print the values of μ and σ used to generate the normal random numbers and also the values of mean and s.d. obtained from the sample drawn from it. Evaluate kurtosis for this selected sample given by $\mu_4 = \frac{k_4}{s^4} - 3$, where s is s.d. of the sample, and k_4 is the fourth order central moment about mean $\left(k_4 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4\right)$ for the sample. Print the value of k_4 and μ_4.

(b) Given $D(T) = \int_0^{\frac{\theta_D}{T}} \frac{x^4 e^x dx}{\left(e^x - 1\right)^2}$ and $\theta_D = 10K$. For T within range 1 K to 50 K evaluate $f(T) = 3\left(\frac{T}{\theta_D}\right)^3 D(T)$ using Monte-Carlo method for evaluating the integral. Plot $f(T)$ vs. T. Label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

8. (a) A single 1 D random walk consists of N steps. Evaluate the square of the distance of the particle from the origin after i steps $\{i = 1(1)\,N\}$) and store the result in an array. Print the last 5 elements of the array. Plot the coordinate of the particle after i steps vs i. Label both axes of the graph. Set a proper title. Save the graph in suitable format. (Take $N = 1000$ as runtime input.)

(b) Evaluate the following Integral using Monte-Carlo method of integration. $\int_0^{10} \frac{xdx}{x^2+16}$. Take the number ($n$) of random points as input. Run the program for two different values of n. Print the results. 10+10

2022

PHYSICS — HONOURS — PRACTICAL

Paper : CC-12P

(Statistical Physics)

Full Marks : 30

The figures in the margin indicate full marks.

Set-III

LNB	:	05
viva voce	:	05
Experiment	:	20

1. (a) Generate N uniform random numbers between 5 and 10, [5, 10). Store them in an array. Evaluate the mean and standard deviation of the numbers generated. Calculate the autocorrelation function $r_k = \frac{c_k}{c_0}$ of the sèries for $k = 1$ and 10. Print the values of mean, standard deviation of the random series, and r_1 and r_{10} for the series. Take N as runtime input ($N = 100$ and $N = 1000$).

(b) Plot Fermi Dirac Distribution $f(E,T) = \frac{1}{e^{\frac{E-E_F}{kT}}+1}$ vs T. Given $E_F = 0.3\ eV$ and $k = 8.617 \times 10^{-5} eV/K$. Take the range of T from 10 to 200 K. Take $E = 0.29$ eV, $E = 0.30\ eV$ and $E = 0.31\ eV$. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

2. (a) Generate N uniform random integer between in the range [0, 9]. Store them in an array. Find the mean m and standard deviation s of the random integers. Compute k_4, the fourth order central moment about mean (given by $k_4 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^4$ for the generated random integers. Define kurtosis as $\mu_4 = \frac{k_4}{s^4} - 3$. Print the values of m, s, k_4 and μ_3.

(b) Plot the MB distribution $f(E,T)$ vs T, where $f(E) = \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{\frac{3}{2}}} \sqrt{E} e^{-\frac{E}{kT}}$. $k = 8.617 \times 10^{-5} eV/K$.

Take the range of T from 100 to 2000 K. Take E = 0.05 eV, E = 0.10 eV and E = 0.15 eV. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format.

10+10

3. (a) Generate N uniform random numbers within [0,1) and store them in an array. Draw n samples from the array, each sample should contain s random numbers. Compute the mean of these n samples and store the means in an array. Plot the original array and the array of means in separate Histogram using subplot. Take N = 100000, n = 1000 and s = 40 as runtime inputs. Comment on the nature of the distribution that the means follow.

(b) Plot the BE distribution $f(E, T)$ vs. T, where $f(E) = \frac{1}{e^{\frac{E-\mu}{kT}} - 1}$. $k = 8.617\times10^{-5}$ eV/K, $\mu = 0.1\ eV$.

Take the range of T from 10 to 400 K. Take $E = 0.12$ eV $E = 0.15$ eV and E = 0.18 eV. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format.

10+10

4. (a) A single trial of a random experiment is tossing an unbiased coin m times. Simulate an experiment consisting of N trials. Plot the total number of heads obtained per trial in histogram. Set a proper title. Save the graph in suitable format. (m = 40 and N = 10000 during runtime.)

(b) Evaluate the following Integral using Monte-Carlo method of integration. $\int_0^{\pi} x^2 \sin\left(\frac{x}{2}\right) dx$. 10+10

5. (a) Considering decay of a given nucleus to be independent of each other and $\lambda \equiv$ probability of decay of a single nucleus per unit time. Simulate radio active decay for N_0 number of parent nuclei at $t = 0$. Consider the final time to be t_n and number of un-decayed nuclei at time t_n be $N(t_n)$. Also evaluate it directly using the standard relation and print the percentage given by

$$100\times\frac{N(t_n)_{simulated} - N(t_n)_{exact}}{N(t_n)_{exact}}.$$ Use $\lambda = 0.004$, $N_0 = 50000$ and $t_n = 470$.

(b) The mean occupation number of a single particle state of energy ϵ is given by $\langle n_\epsilon \rangle = \dfrac{1}{e^{\frac{\epsilon-\mu}{kT}}+1}$ for fermion. Plot $\langle n_\epsilon \rangle$ vs kT. Consider the range of T within 10 to 1000 K. Use $\epsilon = 0.44$ and $\epsilon = 0.46$, for $\mu = 0.45$ eV. Use legend label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

6. (a) A single $2D$ random walk consists of n steps. Simulate a collection of N such random walks. Evaluate the r.m.s. value of end to end distance (r) for different number of steps $(t)t = 1$ to n). Fit the following function with the data $\ln r(t) = A + B \ln t$. Print the values of A and B. Find the value of r at $t = 10$.

(b) Given C_v according to Einstein Theory : $C_v = 3R\dfrac{\left(\frac{\theta_E}{T}\right)^2 e^{\frac{\theta_E}{T}}}{\left(e^{\frac{\theta_E}{T}}-1\right)^2}$. Plot $\dfrac{C_v}{3R}$ vs T for T lying within the range 1 to 50 K for Einstein Theory for $\theta_E = 10$ and $\theta_E = 20\ K$. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format. 10+10

7. (a) Generate N normal random number with mean (μ) and standard deviation (σ). Store it in an array. Evaluate mean and standard deviation for randomly selected 20 numbers from the array. Print the values of μ and σ used generate the normal random numbers and also the values of mean and Standard Deviation(s) obtained from the sample drawn from it. Evaluate skewness for both the array of numbers. $\mu_3 = \frac{k_3}{s^3}$, where $k_3 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^3$.

(b) Given $D(T) = \int_0^{\frac{\theta_D}{T}} \frac{x^4 e^x dx}{(e^x - 1)^2}$. For T within range 1 K to 40 K evaluate $f(T) = 3\left(\frac{T}{\theta_D}\right)^3 D(T)$.

Plot $f(T)$ vs T. Take $\theta_D = 4$K and $\theta_D = 10$ K. Use legend and label the axes of the graph. Set a proper title. Save the graph in suitable format.

10+10

8. (a) A single 1D random walk consists of n steps. Simulate a collection of N such random walks. Evaluate the r.m.s. value of end to end distance (r) for different number of steps (t = 1 to n). Fit the following function with the data $r(t) = at^b$, Print the values of a and b. Plot the fitted function as well as the actual r.m.s values vs t in the same plot. Use legend label both the axes of the graph. Set a proper title. Save the graph in suitable format.

(b) Evaluate the following Integral using Monte-Carlo method of integration. $\int_1^{10} \frac{dx}{x}$. Take the number (n) of random points as input. Run the program for two different values of n. Print the results. Evaluate the value of e from the result of the integration.

10+10

X(6th Sm.)-Physics-H/Pr./(CC-14-P)/
CBCS(Syllabus 2019-2020 & 2018-2019)

2022

PHYSICS — HONOURS — PRACTICAL

(Syllabus : 2018 - 2019)

Paper : CC-14-P

(Statistical Physics)

Full Marks : 30

The figures in the margin indicate full marks.

Distribution of Marks : Program - 20, LNB - 5; viva = 5.

Answer ***any one*** question. 20

1. A radioactive sample of C-14 carbon nuclei has initially 1000 nuclei. Half life time of a C-14 carbon nucleus is $t_{\frac{1}{2}} = 5730$ years.

   (a) Statistically calculate the number of nuclei present in the sample at any time t.

   (b) Plot $N(t)$ vs t graph.

   (c) Also plot the exact theoretical result for radioactive decay in the same graph.

2. (a) Generate 10000 random numbers from uniform random number distribution [0, 1).

   (b) Using appropriate transformation generate exponential distribution $f(x) = \lambda e^{-\lambda x}$ from this uniform distribution.

   (c) Plot the histogram of the exponential distribution so obtained.

   (d) Verify this exponential distribution by plotting exponential function $f(x) = \lambda e^{-\lambda x}$ on the same graph.

3. (a) Using Central Limit Theorem (CLT) generate Gaussian distribution $G(X)$ from uniform distribution [0, 1).

   (b) Print the values of mean and standard deviation of the both uniform distribution and Gaussian distribution.

   (c) Plot the histogram of the Gaussian distribution $G(X)$.

   (d) Check whether this distribution is Gaussian or not by plotting a Gaussian function with appropriate μ and σ.

Please Turn Over

4. Using Monte Carlo Integration method evaluate the following integration $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ with $x = 1$. Plot the function and point accumulated under the curve in this method.

5. A random walker performs a one-dimensional random walk from origin. Draw a trajectory of the walker for 50 random steps (the random walk consists of 50 random steps).
   (a) Calculate the mean and mean square displacement of the random walker for 50 random steps averaged over 3000 independent random walks.
   (b) Plot the mean and mean square displacement vs. the number of steps.
   (c) Determine the slopes of the two curves.
   (d) Calculate the probability of taking n steps to right out of N steps.
   (e) Plot this probability distribution function with n using **plt.bar()** function.

6. Consider a random walker performs two dimensional random walk from the origin.
   (a) Calculate mean square displacement (end to end distance) of the random walker for 50 random steps averaged over 1000 independent random walks (one random walk consists of 50 random steps).
   (b) Plot the mean square displacement vs number of steps and find the slope.

7. Draw the following distribution functions $f(E)$ vs E for three different temperatures.
   (a) Maxwell-Boltzmann distribution function
   (b) Bose-Einstein distribution function
   (c) Fermi-Dirac distribution function.
