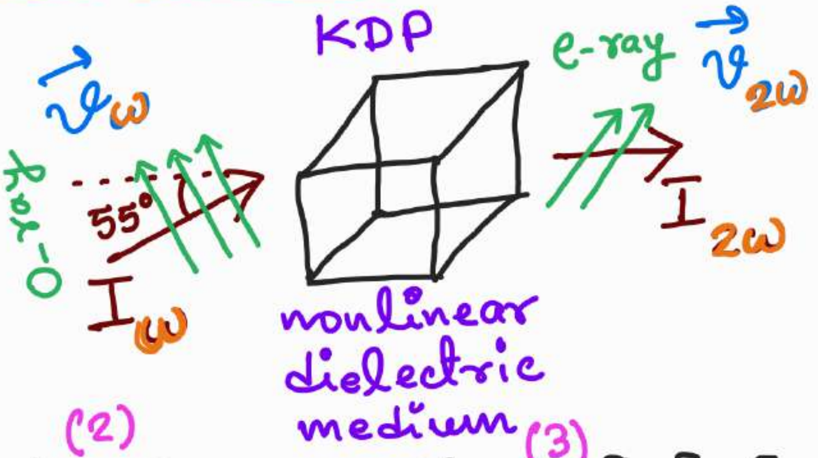
NON LINEAR OPTICS

polarization

free space permittivity

$P = \epsilon_0 \chi E$ — electric field

electric polarizability



$$P_i = \epsilon_0 \Big[\underbrace{\chi^{(1)}_{ij}}_{\substack{9 \text{ component} \\ \equiv 6 \text{ independent component}}} E_j + \underbrace{\chi^{(2)}_{ijk}}_{\substack{27 \text{ component} \\ \equiv 18 \text{ independent component}}} E_j E_k + \underbrace{\chi^{(3)}_{ijkl}}_{\substack{81 \text{ component} \\ \equiv 54 \text{ independent component}}} E_j E_k E_l + \cdots \Big]$$

\# isotropic sample: $P = \epsilon_0 \left(\chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \cdots \right)$

$E = E_0 \sin\omega t$ (say) $\Rightarrow$

$$\epsilon_0 \chi^{(1)} E_0 \sin\omega t + \epsilon_0 \chi^{(2)} E_0^2 \sin^2\omega t + \epsilon_0 \chi^{(3)} E_0^3 \sin^3\omega t + \cdots$$

$$= \epsilon_0 \chi^{(1)} E_0 \sin\omega t + \frac{\epsilon_0 \chi^{(2)}}{2} E_0^2 (1 - \cos 2\omega t) + \frac{\epsilon_0 \chi^{(3)}}{4} E_0^3 (3\sin\omega t - \sin 3\omega t)$$

dc polarization (optical rectification)

frequency doubling / 2nd harmonic generation (SHG)

NO SHG

① Has center of symmetry / inversion center (calcite)

② Not Piezoelectric. quartz, KDP, ADP $\Rightarrow$ Piezoelectric

maximum $I_{2\omega}$

$$I_{2\omega} \propto \sin^2\left[\frac{2\pi (n_\omega - n_{2\omega}) l}{\lambda_0} \right] \Rightarrow l_c = \frac{\lambda_0}{4|n_\omega - n_{2\omega}|}$$

$I_{2\omega} \ldots \dfrac{\ldots}{(n_\omega - n_{2\omega})^2}$ coherence length

Index Matching: $n_\omega = n_{2\omega}$

Frequency mixing: $E = E_{01}\sin\omega_1 t + E_{02}\sin\omega_2 t$

$$\Rightarrow P = \chi^{(1)}\text{term} + \epsilon_0\chi^{(2)}\left(E_{01}^2\sin^2\omega_1 t + E_{02}^2\sin^2\omega_2 t + 2E_{01}E_{02}\sin\omega_1 t\sin\omega_2 t\right) + \cdots$$

$$= \chi^{(1)}\text{term} + \frac{\epsilon_0\chi^{(2)}}{2}E_{01}^2(1-\underbrace{\cos 2\omega_1 t}_{\text{SHG}}) + \frac{\epsilon_0\chi^{(2)}}{2}E_{02}^2(1-\underbrace{\cos 2\omega_2 t}_{\text{SHG}}) + \epsilon_0\chi^{(2)}\left[\cos(\underbrace{\omega_1-\omega_2}_{\text{difference photon}})t - \cos(\underbrace{\omega_1+\omega_2}_{\text{sum photon}})t\right]$$

Maxwell's eq^n: for linear, homogeneous, isotropic medium:

① $\bar{\nabla}\cdot\bar{D} = \rho \xleftrightarrow{\text{Gauss's law}} \oiint_S \bar{D}\cdot d\bar{S} = \iiint_V \rho\, dV$

② $\bar{\nabla}\cdot\bar{B} = 0 \longleftrightarrow \oiint_S \bar{B}\cdot d\bar{S} = 0$

③ $\bar{\nabla}\times\bar{E} = -\dfrac{\partial\bar{B}}{\partial t} \xleftrightarrow[\text{Lenz's law}]{\text{Faraday's law}} \oint_C \bar{E}\cdot d\bar{l} = -\iint_S \dfrac{\partial\bar{B}}{\partial t}\cdot d\bar{S}$

④ $\bar{\nabla}\times\bar{H} = \bar{J}_c + \dfrac{\partial\bar{D}}{\partial t} \xleftrightarrow[\text{Ampere's law}]{} \oint_C \bar{H}\cdot d\bar{l} = \iint_S \Big(\underbrace{\bar{J}_c}_{\text{Ampere}} + \underbrace{\dfrac{\partial\bar{D}}{\partial t}}_{\text{Maxwell}}\Big)\cdot d\bar{S}$

$\bar{\nabla}\cdot\bar{J}_c + \dfrac{\partial\rho}{\partial t} = 0 \to$ continuity

$\bar{\nabla}\cdot$④ $\Rightarrow 0 = \bar{\nabla}\cdot\bar{J}_c + \dfrac{\partial}{\partial t}\bar{\nabla}\cdot\bar{D} = -\dfrac{\partial\rho}{\partial t} + \dfrac{\partial}{\partial t}\bar{\nabla}\cdot\bar{D}$

$\therefore \bar{\nabla}\cdot\bar{D} = \rho - C$, where $C \neq C(t)$. But

$\therefore\ \bar{\nabla}\cdot\bar{D} - \rho = C_1$ ... $C_1 \neq C_1(t)$, but

$C_1 = 0$ otherwise $\oint\!\!\oint_S \bar{D}\cdot d\bar{S} = \iiint_V \rho\, dV + C_1 V$, so

$C_1 V$ acts as source which cannot be true. $\therefore C_1 = 0$

\# $\bar{\nabla}\cdot(3) \Rightarrow 0 = -\dfrac{\partial}{\partial t}\bar{\nabla}\cdot\bar{B}$ $\therefore \bar{\nabla}\cdot\bar{B} = C_2$ where

$C_2 \neq C_2(t)$ but then $\oint\!\!\oint_S \bar{B}\cdot d\bar{S} = C_2 V$ acts as source which cannot be true $\therefore C_2 = 0$.

Basically (1) & (2) are subset of (3) & (4), so 6 scalar equations for 12 unknowns $(\bar{E}, \bar{B}, \bar{D}, \bar{H})$, so 6 more equations (constituitive equations $\bar{D} = \epsilon\bar{E}$, $\bar{B} = \mu\bar{H}$) are required.

$$\bar{D} = \epsilon\bar{E} = \epsilon_0\bar{E} + \bar{P} = \epsilon_0(1+\chi)\bar{E} = \epsilon_0\epsilon_r \bar{E}$$

relative permeability

\# $\bar{\nabla}\times(3) \Rightarrow \bar{\nabla}\times\bar{\nabla}\times\bar{E} = -\dfrac{\partial}{\partial t}\bar{\nabla}\times\bar{B} = -\mu\dfrac{\partial}{\partial t}\bar{\nabla}\times\bar{H}$

$(\because \bar{B} = \mu\bar{H})$

$$\text{or, } \bar{\nabla}(\bar{\nabla}\cdot\bar{E}) - \nabla^2\bar{E} = -\mu\left[\frac{\partial \bar{J}_c}{\partial t} + \frac{\partial^2 \bar{D}}{\partial t^2}\right] = -\mu\left[\frac{\partial \bar{J}_c}{\partial t} + \epsilon\frac{\partial^2 \bar{E}}{\partial t^2}\right]$$

$$\text{or } \nabla^2\bar{E} = \mu\left[\frac{\partial \bar{J}_c}{\partial t} + \epsilon\frac{\partial^2 \bar{E}}{\partial t^2}\right] + \nabla\left(\frac{\rho}{\epsilon}\right) \quad \left(\because \bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon}\right)$$

$$\text{or } \nabla^2\bar{E} = \mu\frac{\partial \bar{J}_c}{\partial t} + \mu\epsilon\frac{\partial^2 \bar{E}}{\partial t^2} + \frac{1}{\epsilon}\bar{\nabla}\rho$$

$$\text{or } \nabla^2\bar{E} \simeq \mu\epsilon\frac{\partial^2 \bar{E}}{\partial t^2} \quad (\text{In free space } \bar{J}_c = \rho = 0)$$

Similarly $\nabla^2 \bar{H} = \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2} = v^{-2} \frac{\partial^2 \bar{H}}{\partial t^2}$, $v = \frac{1}{\sqrt{\mu\epsilon}}$

Wave equation solution in Cartesian, cylindrical (Bessel's fn), Spherical polar. coordinate system.

Anisotropic medium: $D_i = \epsilon_{ij} E_j$; $\epsilon_{ij} = \epsilon_{ji}$